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Short Note

A counterexample to a conjecture of Jafari Rad and Volkmann

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Abstract: In this short note, we disprove the conjecture of Jafari Rad and Volkmann that every γ-vertex critical graph is γ_R-vertex critical, where $\gamma(G)$ and $\gamma_R(G)$ stand for the domination number and the Roman domination number of a graph G, respectively.

Keywords: Roman domination, γ -vertex critical graphs, γ_R -vertex critical graphs

AMS Subject classification: 05C69

1. Introduction

For a graph $G = (V, E)$, let $\gamma(G)$ and $\gamma_R(G)$ denote the domination number and the Roman domination number of G, respectively. For $\mu \in \{\gamma, \gamma_R\}$, a graph G is said to be μ-vertex critical if removing any vertex of G decreases $\mu(G)$, that is $\mu(G - v) < \mu(G)$ for every vertex $v \in V$. The study of γ-vertex critical graphs was initiated by Brigham et al. [1] while the study of γ_R-vertex critical graphs was initiated by Hansberg et al. [3]. Jafari Rad and Volkmann continued the study of γ_R-vertex critical graphs in [4], where they posed the following conjecture. It is worth noting that this conjecture also appears in the recent book chapter on Roman domination (see [2], pp 393).

Conjecture 1. ([4]) Any γ-vertex critical graph is γ_R-vertex critical.

We note that it was shown in [4] that the converse of Conjecture 1 was not true by giving an example of γ_R-vertex critical graphs that are not γ-vertex critical. The counterexample that we provide to Conjecture 1 is in fact a family of connected

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cactus graphs G_n obtained by $n \geq 2$ disjoint cycles C_4 sharing a same vertex which we denote by v (for example, see the graph G_4 shown in Figure 1). It is a simple matter to check that $\gamma(G_n) = n + 1$ and $\gamma_R(G_n) = n + 2$. Moreover, G_n is γ-vertex critical since $\gamma(G_n - x) = n$ for every vertex x of G_n. Nevertheless for vertex v, $\gamma_R(G_n - v) = 2n \geq \gamma_R(G_n) = n + 2$, and therefore G_n is not a γ_R-vertex critical graph.

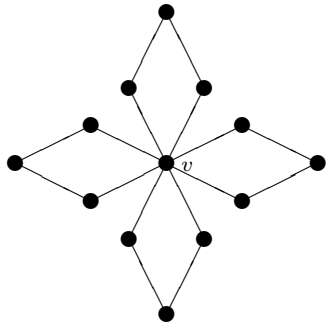

Figure 1. **Connected cactus graph** G_4

References

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