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Short Note

A note on Roman k-tuple domination number

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Abstract: For an integer $k \geq 2$, a Roman k-tuple dominating function, (or just RkDF), in a graph G is a function $f\colon V(G) \to \{0,1,2\}$ satisfying the condition that every vertex u for which $f(u) = 0$ is adjacent to at least k vertices v for which $f(v) = 2$, and every vertex u for which $f(u) \neq 0$ is adjacent to at least $k-1$ vertices v for which $f(v) = 2$. The Roman k-tuple domination number of G is the minimum weight of an RkDF in G. In this note we settle two problems posed in [Roman k-tuple Domination in Graphs, Iranian J. Math. Sci. Inform. 15 (2020), 101–115].

Keywords: Roman domination, Total Roman domination; Roman k-tuple domination

AMS Subject classification: 05C69

1. Introduction

For a graph $G = (V, E)$, we denote by $N(v)$ the *open neighborhood* of a given vertex v. A function $f : V \longrightarrow \{0, 1, 2\}$ is called a *Roman dominating function* or just an RDF if for every vertex $v \in V$ with $f(v) = 0$, there exists a vertex $u \in N(v)$ with $f(u) = 2$. The *weight* of an RDF f is the sum $f(V) = \sum_{v \in V} f(v)$. The minimum weight of an RDF on G is called the *Roman domination number* of G and is denoted by $\gamma_R(G)$. A *total Roman dominating function* (TRDF) of a graph G with no isolated vertex is an RDF f such that the subgraph induced by $\{v : f(v) \neq 0\}$ has no isolated vertex. The *total Roman domination number* $\gamma_{tR}(G)$ is the minimum weight of a TRDF on G. The concept of Roman domination was first studied in depth by Cockayne et al. [3], and subsequently developed with many variations, see for example, [1, 2]. Recently, Kazemi [4] introduced Roman k-tuple dominating functions. For an integer $k \geq 2$, a *Roman k-tuple dominating function*, (or just RkDF), in a graph G is a function $f\colon V(G) \to \{0, 1, 2\}$ satisfying the condition that every vertex u for which $f(u) = 0$

is adjacent to at least k vertices v for which $f(v) = 2$, and every vertex u for which $f(u) \neq 0$ is adjacent to at least $k-1$ vertices v for which $f(v) = 2$. The *Roman k-tuple domination number*, denoted $\gamma_{\times k,R}(G)$, is the minimum weight of an RkDF in G. Kazemi in [4] posed the following problems.

Problem 1. Characterize graphs G with $\gamma_{\times 2,R}(G) = \gamma_R(G)$.

Problem 2. Find graphs G with $\gamma_{\times 2,R}(G) = \gamma_{tR}(G)$.

For an RDF (TRDF, or R2DF) f we denote $f = (V_0, V_1, V_2)$, where $V_i = \{v \in V : f(v) = i\}$, for $i = 0, 1, 2$. A R2DF with minimum weight is called a $\gamma_{\times 2,R}(G)$-function. Let G be a graph of order $n \geq 2$, and let $f = (V_0, V_1, V_2)$ be a $\gamma_{\times 2,R}(G)$-fuction. Clearly $|V_2| \geq 2$. Let $v \in V_2$. Then $g = (V_0, V_1 \cup \{v\}, V_2 - \{v\})$ is a TRDF for G, and $h = (V_0 \cup \{v\}, V_1, V_2 - \{v\})$ is an RDF for G. Thus, $\gamma_{\times 2,R}(G) \geq \gamma_{tR}(G) + 1$ and $\gamma_{\times 2,R}(G) \geq \gamma_R(G) + 2$. Both bounds are sharp, as can be seen in a cycle C_3. Thus, there is no graph G with $\gamma_{\times 2,R}(G) = \gamma_R(G)$ or $\gamma_{\times 2,R}(G) = \gamma_{tR}(G)$, and so both Problems 1 and 2 are settled.

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