

## Cost, Revenue and Profit Efficiency in multi-period network system: A DEA-R based Approach

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**Abstract:** It has been proven that Data Envelopment Analysis is an efficient method to compare different decision making units with multiple inputs and outputs, but traditional Data Envelopment Analysis models suffers some difficulties: (a)- the inputs and outputs are not supposed to be given in terms of ratio. Thus, when the data are partially available, the decision maker will be unable to access missing data from the present data; (b) in measuring the efficiency of a set of decision making units for some periods, the conventional Data Envelopment Analysis based technique cannot handle the problem posed in a periodic form where the costs, profits and revenue efficiency of the main problems in the network structures are required. The contribution of this paper is four folded: (1) the cost, revenue and profit efficiency of each stages are calculated from the proposed method depends on the performance of the unit in both stages. (2) Our method evaluates the total cost, revenue and profit efficiency in a whole  $t$  ( $t = 1, \dots, T$ ) time periods derived from all periodic and every stage efficiency, (3) The proposed method in this study yields the efficiency measures deals with ratio data, (4) To elucidate the details of the proposed method, the proposed multi-period DEA-R method was employed to measure the efficiency of ten units in three separate time periods. Numerical examples are also provided to explain the presented methods.

**Keywords:** Ratio Data Envelopment Analysis (DEA-R), Network multi-Periodic Production Process, overall efficiency, cost- Revenue and profit efficiency

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## 1. Introduction

Data Envelopment analysis (DEA) is concerned with comparative assessment of the efficiency of decision making units (DMUs). The classical DEA models, maximize the efficiency of a system as a block box without considering the internal structure. The idea of evaluating a system including the internal structures was pioneered by [7] in 1996 and called as Network Data Envelopment Analysis (NDEA). Since then, many researchers have investigated on the NDEA and have used different models with various practical applications. For cases in which the period of time being examined is composed of clearly defined time units such as years, classical DEA models suffers from evaluation problems. In this multi-period cases, the total inputs consumed and total outputs produced in all periods are aggregated for efficiency measurement [12]. To deal with this concern, Seifford and Zhu [23] and Lu [15] presented two-stage model to study the inter-relations of consecutive periods in efficiency analysis and evaluated efficiency frontier. Cook et al. [2] have studied different pitfalls and gaps in multi-stage network structures. Next, Chen et al. [1] have employed the NDEA to determine the efficiency of sub-processes and overall efficiency. Kao and Huang [14] developed a multiplicative model to calculate network efficiency. In their model, the weights of the inputs and outputs are achieved in such a way that the efficiency of the network is maximized provided that the efficiency of each stage does not exceed unity. Tone and Tsutsui [25] proposed a network Slacks-Based measure (NSBM) for sequential network structures. Pourmahmoud [22] introduced a new model for ranking DMU based on Dynamic DEA. Omrani and Soltanzadeh [21] proposed a relational dynamic NDEA (DNDEA) model which measures the efficiencies of the system and its internal processes over the time, simultaneously for evaluating the performance of a DMU with interrelated processes during specified multiple periods. Esmailzadeh and Kazemi Matin [5] extended multi-period DEA models by considering more complex internal relations for the sub-processes of each decision making unit, DMU. They presented novel multi-period network DEA models that were developed for performance evaluation of overall and specific time period efficiencies with parallel and series internal structures in the sub-processes for each time period. Gazari Neishaburi et al. [9] proposed a dynamic data envelopment analysis model that measures the process efficiency in a real business. Their proposed dynamic NDEA model is fully designed and customized for IMI which is a leading institute in providing consulting management, publication, and educational services. Esfidani et al. [4] used a non-radial DEA model called the network slacks-based measure (NSBM) model to measure the efficiency of a system with a multi-period two-stage structure. Then they described the properties of the proposed model in details. Moreover, they decomposed the overall efficiency of the system over a number of time periods as a weighted average of the efficiency in each period. Najari Alamuti et al. [20] for the first time, a systematic approach used to present a dynamic NDEA. A common perspective in all above mentioned studies was analyzing overall and periodic efficiencies. Every paper has been focused on different features of network structure and different techniques were challenged. However,

the relation between the specific-periodic efficiency and the overall efficiency arise as a question. One of the solution to this problem is employing DEA-R models. Ratio-based DEA (DEA-R) models were first formulated in [3]. This technique was introduced as a tool that combines DEA and ratio analysis, and since then, such models have been studied and applied by many other researchers. Wei et al. [26–28] has focused on relations between traditional DEA models and ratio-based DEA-R models and applied the DEA-R models for an efficiency analysis of 21 medical centers in Taiwan. Mozaffari et al. [19] discussed cost and revenue efficiency in DEA and DEA - R models and the reviewed the relationship between DEA models without explicit input and DEA-R. Mozaffari et.al. [18] also discussed the axioms for determining the production possibility set in constant returns to scale technology for DEA-R, and, finally an original algorithm for identification of efficient surfaces in this class of models is proposed. For more references, the reader may refer to [11, 16, 17]. Looking closely to the problem of network structures, the question of cost, revenue and profit in multi-period network structures has attracted many researchers. In order to assessment of the cost, revenue and profit efficiency in multi-period network structures, Tohidi and Tohidnia [24] employed DEA-R models providing linear models. The advantage of the proposed profit efficiency model using the DEA-R compared to the existing models was its flexibility which followed by the opinions of decision makers. Another advantage was the calculation of the maximum possible profit using revenue-cost ratios and calculates Malmquist productivity index. Finally, DEA-R profit efficiency score obtained by this method was always a positive value. Ghiyasi et al. [10] proposed the parallel cost efficiency model to utilize the operation of internal processes for measuring cost efficiency of the system. They proposed a cost efficiency decomposition of parallel systems, where each process utilized external inputs to produce external outputs. In addition to the theoretical aspect, as an empirical contribution, they implemented the proposed model for cost efficiency analysis of hospitals in Mashhad for two levels, i.e. hospital level and ward level. Fare and Zelenyuk [8] derived a new theoretical result that showed the Nerlovian profit efficiency is a special case of the recently introduced general profit efficiency measure. They also presented a new decomposition of profit efficiency. Finally, They also outlined a simple way of estimating profit efficiency in the Data Envelopment Analysis (DEA) and Free Disposal Hull (FDH) frameworks, while avoiding the computational intensity of linear programming and circumventing the lack of more detailed data. Fallahnejad and RezaeiHezaveh [6] aimed at providing new estimation of cost efficiency and sources of losses in the total efficiency in a non-competitive environment where there is the possibility of change in prices of inputs and outputs from one DMU to another. Jamasb Ghalati et.al [13] has presented a model for evaluation cost and revenue efficiency in presence of controllable and uncontrollable indicator based on DEA-R method. They also showed that the DEA-R based model outperformed the cost and revenue efficiency evaluation based on DEA.

The aim of this paper is to develop a multi-period production system, based on the DEA-R approach, to measure the cost, revenue and profit efficiency of a set of DMUs

in a period of time. To do so, two-phased procedure is considered: First phase, employing DEA-R model in the whole system efficiencies (cost, profit and revenue) are evaluated. Because of the presence of the zero-weights, some inputs may not play a role in producing some outputs. To overcome the problem, the model is developed based on the weight concepts in multi-periodic system. Second phase, the periodic efficiencies has been evaluated for each separate time span. The overall cost, revenue and profit efficiency of the multi-period network system is evaluated as the weighted average of periodic cost, revenue and profit efficiencies in each time span. The paper is organized as follows. The following section reviews the basic concepts of multi-period production system. A brief summary of DEA-R is presented in Section 3, then the proposed approach for dealing with multi-period system based on DEA-R models for evaluating cost, profit and revenue efficiencies is introduced. Section 4 illustrates the applicability of the proposed method with a real numerical example. Conclusion will end the paper.

## 2. Preliminaries

### 2.1. DEA models for cost, revenue and profit efficiency

Evaluating efficiency in multi-period models has attracted considerable attention among researchers. To describe the DEA efficiency measurement, assume that there are  $n$  DMUs,  $DMU_j$  ( $j = 1, \dots, n$ ) and the performance of each DMU is characterized by a production process of  $m$  inputs  $X_j = (x_{1j}, x_{2j}, \dots, x_{mj}) \in R^m$  to produce outputs  $Y_j = (y_{1j}, y_{2j}, \dots, y_{sj})^T \in R^s$ . Also,  $Y_j \geq 0$ ,  $Y_j \neq 0$  and  $X_j \geq 0$ ,  $X_j \neq 0$ . Suppose that  $C = (c_1, c_2, \dots, c_m)^T \in R^m$  is a specific vector of input costs. The production possibility set (PPS) for CCR model is defined as follows:

$$T = \{(X, Y) \mid X \geq \sum_{j=1}^n \lambda_j X_j, Y \leq \sum_{j=1}^n \lambda_j Y_j, \lambda_j \geq 0, j = 1, \dots, n\} \quad (1)$$

Regarding the above PPS, the cost efficiency of under evaluation unit,  $DMU_o$  is evaluated employing the following model as follows.

$$\begin{aligned} \text{Min } W &= \sum_{i=1}^m c_i s_i \\ &\sum_{j=1}^n \lambda_j x_{ij} \leq s_i, i = 1, \dots, m \\ &\sum_{j=1}^n \lambda_j y_{rj} \leq y_{ro}, r = 1, \dots, s \\ &\lambda_j \geq 0 \end{aligned} \quad (2)$$

Employing the optimal solution of model (2)  $W^*$ , the cost efficiency for each unit is defined as  $E_{co} = \frac{W^*}{\sum_{i=1}^m c_i x_{io}} \leq 1$ . Clearly, the amount of cost efficiency does not exceed

unity. Given the output price vector  $p = (p_1, \dots, p_r)^T \in R^s$  the revenue efficiency is evaluated employing the following model

$$\begin{aligned}
 Max \ W &= \sum_{r=1}^s p_r t_r \\
 &\sum_{j=1}^n \lambda_j x_{ij} \leq x_{io}, i = 1, \dots, m \\
 &\sum_{j=1}^n \lambda_j y_{rj} \geq t_{ro}, r = 1, \dots, s \\
 &\lambda_j \geq 0
 \end{aligned} \tag{3}$$

Regarding to the optimal solution of model (3),  $(W^*, \lambda^*)$ , the revenue efficiency of  $DMU_o$  is evaluated as follows  $E_{po} = (\sum_{r=1}^s p_r y_{ro})/W^* \leq 1$ . Given the both output price vector and input cost vector as  $p = (p_1, \dots, p_r)^T \in R^s$  and  $C = (c_1, c_2, \dots, c_m)^T \in R^m$ , respectively, another efficiency named as profit efficiency is assessed applying the following mathematical programming:

$$\begin{aligned}
 Max \ W &= \sum_{r=1}^s p_r t_r - \sum_{i=1}^m c_i s_i \\
 &s_i = \sum_{j=1}^n \lambda_j x_{ij} \leq x_{io}, i = 1, \dots, m \\
 &t_{ro} = \sum_{j=1}^n \lambda_j y_{rj} \geq y_{ro}, r = 1, \dots, s \\
 &\lambda_j \geq 0
 \end{aligned} \tag{4}$$

Having obtained the optimal solution of model (4) the profit efficiency is as follows:

$PE = \frac{\sum_{r=1}^s p_r y_{ro} - \sum_{i=1}^m c_i x_{io}}{W^*} \leq 1$ . Clearly,  $0 < PE \leq 1$  and  $DMU_o$  is profitable if and only if the objective function of model (4) achieves unity in optimality.

## 2.2. DEA-R models for cost, revenue and profit efficiency

According to the cost vector  $C = (c_1, c_2, c_3, \dots, c_m)^T$  and the price vector  $p = (p_1, p_2, p_3, \dots, p_s)^T$  Mozaffari et al. [19] two models (5) and (6) presented for cost and revenue in DEA-R:

$$\begin{aligned}
 \min \ W_R &= \sum_{i=1}^m c_i x_i \\
 S.t. \ &\sum_{j=1}^n \lambda_j \left( \frac{x_{ij}}{y_{rj}} \right) \leq \frac{x_i}{y_{ro}}, i = 1, \dots, m \\
 &\sum_{j=1}^n \lambda_j = 1, r = 1, \dots, s \\
 &\lambda_j \geq 0, j = 1, \dots, n
 \end{aligned} \tag{5}$$

$$\begin{aligned}
\max \psi_R^* &= \sum_{r=1}^s p_r y_r \\
\text{S.t.} \quad &\sum_{j=1}^n \lambda_j \left( \frac{y_{rj}}{x_{ij}} \right) \geq \frac{y_r}{x_{io}}, i = 1, \dots, m \\
&\sum_{j=1}^n \lambda_j = 1, r = 1, \dots, s \\
&\lambda_j \geq 0, j = 1, \dots, n
\end{aligned} \tag{6}$$

In model (5), it can be seen that the cost model in DEA-R is based on the ratio of input to output. Considering the first clause of model (5), we have

$$\sum_{j=1}^n \lambda_j \left( \frac{x_{ij}}{y_{rj}} \right) \leq \frac{x_i}{y_{ro}}$$

That is, the possibility of producing an output less than  $y_{ro}$  is modeled by considering the variable  $x_i$  for inputs to outputs. Also, considering the value of the objective function of the model (5), the minimum input for  $DMU_o$  is considered as minimization, provided that it applies within its constraints.

They defined the cost and revenue efficiency of a decision unit  $DMU_o$  of the DEA-R model by models (7) and (8), which is always less than or equal to one:

$$CE_{DEA-R} = \frac{C^T X^*}{C^T X_o} = \frac{W^*}{C^T X_o} \leq 1 \tag{7}$$

It is clear that the value of  $CE_{DEA-R_o}$  is always less than or equal to one. DMU is called cost efficient if and only if  $CE = 1$  [19]

$$RE_{DEA-R_o} = \frac{p^T y^*}{p^T y_o} = \frac{\psi^*}{p^T y_o} \geq 1 \tag{8}$$

It is clear that the value of  $RE_{DEA-R_o}$  is always greater than or equal to one. We call the  $DMU_o$  unit efficient if and only if  $\psi^* = 1$  Tohidnia and Tohidi [24] proposed the following model to determine profit in DEA-R:

$$\begin{aligned}
P_R^* &= \max \frac{\sum_{r=1}^s p_r y_r}{\sum_{i=1}^m c_i x_i} \\
\text{S.t.} \quad &\frac{y_r}{x_{io}} = \sum_{j=1}^n \lambda_j \left( \frac{y_{rj}}{x_{ij}} \right) \geq \frac{y_{ro}}{x_{io}}, i = 1, \dots, m, r = 1, \dots, s \\
&\frac{x_i}{y_{ro}} = \sum_{j=1}^n \mu_j \left( \frac{x_{ij}}{y_{rj}} \right) \leq \frac{x_{io}}{y_{ro}}, i = 1, \dots, m, r = 1, \dots, s \\
&\sum_{j=1}^n \lambda_j = 1, \\
&\sum_{j=1}^n \mu_j = 1, \\
&\lambda_j \geq 0, \mu_j \geq 0, j = 1, \dots, n
\end{aligned} \tag{9}$$

Model (9) estimates the maximum accessible profit in respect of the feasible ratios of inputs and outputs rather than input-output levels. Model (9) is formulated by using the ratios created from all the inputs and outputs instead of the original set of inputs and outputs. ([24]). Tohidnia and Tohidi [24] presented the following model to calculate the profit efficiency of  $DMU_o$  in DEA-R:

$$PE_R(x_o, y_o) = \frac{py_o/cx_o}{P_R^*} \quad (10)$$

The form  $PE_R(x_o, y_o)$  is the observed profit and its denominator shows the maximum possible profit that  $DMU_o$  can obtain in the production process. A DMU in DEA-R is profit efficient if and only if  $PE_R(x_o, y_o) = 1$  ([24]).

### 3. Proposed Approach for Efficiency Assessment

In this section the CCR model consists of ratio-based data are stated. Then we introduce the development of this model on multi-period systems as shown in Fig1.

#### 3.1. DEA-R input oriented CCR model

Again suppose that there are  $n$  DMUs, and for  $DMU_j (j = 1, \dots, n)$  the observed data of inputs and outputs are  $X_j = (x_{1j}, \dots, x_{mj}) > 0$  and  $Y_j = (y_{1j}, \dots, y_{sj}) > 0$ . Also assuming the ratios and are defined. Despic et al. [3] have introduced their DEA-R efficiency model for evaluation of  $DMU_o$  under the assumption of constant returns to scale technology as follows:

$$\hat{e}_o = \underset{w_{ir}}{\text{Max}} \underset{j}{\text{Min}} \sum_{i=1}^m \sum_{r=1}^s w_{ir} \frac{\frac{x_{ij}}{x_{io}}}{\frac{y_{rj}}{y_{ro}}} \quad (11)$$

$$s.t. \sum_{i=1}^m \sum_{r=1}^s w_{ir} = 1$$

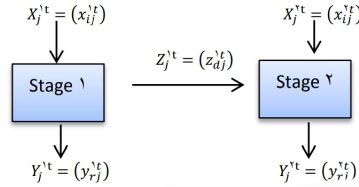
$$w_{ir} \geq 0, r = 1, \dots, s, i = 1, \dots, m \quad (12)$$

The model assumes that  $x_{io}$  and  $y_{ro}$  are the input and output vector of under evaluated unit, and  $w_{ir}$  represents the relative weight of  $i$ -th input and  $r$ -th output of input and output vector variables.

#### 3.2. DEA-R Model in Multi Period Production Process

In this section a multi-period decision process is introduced within which the stages are connected with intermediate measures. Consider a two-stage production process as shown in Fig.1., where the subscript  $t = 1, \dots, T$  denotes the corresponding time period. Suppose again that there are  $n$  DMUs and for the first stage of  $DMU_o$ , the

observed data on the vectors of inputs, final outputs and intermediate-product are denoted as  $X_j^{1t} = \{x_{ij}^{1t}, i \in I(1)\}$  and or  $Y_j^{1t} = \{y_{rj}^{1t}, r \in O(1)\}$  and  $Z_j^{1t} = \{z_{mj}^{1t}, m \in M(1)\}$ . The intermediate measure  $Z_j^{1t}$  ( $j = 1, \dots, n, t = 1, \dots, T$ ) plays a dual role in this structure. The second stage fed up with  $\{x_{ij}^{2t}, i \in I(2)\}$  and the intermediate measure  $Z_j^{1t} = \{z_{mj}^{1t}, m \in M(1)\}$  to produce the final output  $Y_j^{2t} = \{y_{rj}^{2t}, r \in O(2)\}$ . In what follows, an alternative model is introduced to calculate the cost, profit and revenue efficiency of a multi-period production process based on DEA-R models. To do so, we toy around both DEA-R models and the related models for multi-period systems. In order to analyze the multi-periodic systems with DEA-R models, it is desired to measure the cost, profit and revenue efficiency with regards to periodic efficiency and achieve the corresponding optimal weights for each DMU. As stated before, the input price vector for the external inputs in first and second stage are denoted as  $C_j^{1t} = \{c_{ij}^{1t}, i \in I(1)\}$  and  $C_j^{2t} = \{c_{ij}^{2t}, i \in I(2)\}$ , respectively. The final output price vector for the corresponding stages are represented as  $V_j^{1t} = \{v_{rj}^{1t}, r \in O(1)\}$  and  $V_j^{2t} = \{v_{rj}^{2t}, r \in O(2)\}$ , respectively. Regarding to the dual role of the intermediate measure  $Z_j^{1t}$  ( $j = 1, \dots, n, t = 1, \dots, T$ ), i.e., the output of the first stage and the input of the second stage, so, it seems to be rational the price of intermediate product set as zero. Without loss of generality, also, assume that the ratio of  $\left(\frac{x_{ij}^{1t}}{z_{mj}^{1t}}, i \in I(1), m \in M(1)\right)$   $\left(\frac{z_{mj}^{1t}}{y_{rj}^{2t}}, m \in M(1), r \in O(2)\right)$  and  $\left(\frac{x_{ij}^{2t}}{y_{rj}^{2t}}, i \in I(2), r \in O(2)\right)$  are available in each time period  $t = 1, \dots, T$ . Equipped with these assumption, the proposed cost efficiency



**Figure 1.** Two-stage network structure at time  $t(1 \leq t \leq T)$

model has the following format:

$$\begin{aligned}
 & \min \sum_{i \in I(1)} c_i^t f_{i1}^t + \sum_{i \in I(2)} c_i^t f_{i2}^t \\
 & \text{s.t. } \sum_{j=1}^n \lambda_j^{1t} \left( \frac{x_{ij}^{1t}}{y_{rj}^{1t}} \right) \leq \left( \frac{f_{i1}^t}{y_{ro}^{1t}} \right), i \in I(1), r \in O(1) \\
 & \sum_{j=1}^n \lambda_j^{2t} \left( \frac{x_{ij}^{2t}}{y_{rj}^{2t}} \right) \leq \left( \frac{f_{i2}^t}{y_{ro}^{2t}} \right), i \in I(2), r \in O(2) \\
 & \sum_{j=1}^n \lambda_j^{1t} \left( \frac{x_{ij}^{1t}}{z_{mj}^{1t}} \right) \leq \left( \frac{f_{i1}^t}{z_{mo}^{1t}} \right), i \in I(1), m \in M(1)
 \end{aligned}$$



$$\begin{aligned}
\sum_{j=1}^n \lambda_j^{2t} \left( \frac{z_{mj}^{1t}}{y_{rj}^{2t}} \right) &\leq \left( \frac{z_{mo}^{1t}}{y_{ro}^{2t}} \right), m \in M(1), r \in O(2) \\
\sum_{j=1}^n \lambda_j^{kt} &= p_k, 1 \leq k \leq 2 \\
\sum_{k=1}^2 p_k &= 1 \\
\lambda_j^{kt} &\geq 0; \forall 1 \leq j \leq n; 1 \leq k \leq 2
\end{aligned} \tag{13}$$

Notably, the optimal objective function in model (13) calculates the overall cost efficiency with regards to periodic efficiency of all DMUs in all periods and stages. The first constraint in model (13) measures the radial decrease in the ratio of external inputs to final outputs in the first stage. The second constraint in model (13) also measures the radial decrease in the ratio of external inputs to intermediate measures in the second stage. Cost efficiency for the two-stage network structure as shown in Figure 1 is computed as  $C_o^T E = \frac{\sum_{i \in I(1)} f_{i1}^t + \sum_{i \in I(2)} f_{i2}^t}{\sum_{i \in I(1)} x_{ij}^{1t} + \sum_{i \in I(2)} x_{ij}^{2t}}$ . A closer examination of model (13) indicates that the third and fourth constraints can handle the intermediate measure. In order to extend the proposed two stage model and to compute the cost efficiency in  $t(t = 1, \dots, T)$  time periods, model (13) can be developed as model (14).

$$\begin{aligned}
\min \sum_{t=1}^T &\left( \sum_{i \in I(1)} c_i^t f_{i1}^t + \sum_{i \in I(2)} c_i^t f_{i2}^t \right) \\
\text{s.t. } \sum_{j=1}^n \lambda_j^{1t} &\left( \frac{x_{ij}^{1t}}{y_{rj}^{1t}} \right) \leq \left( \frac{f_{i1}^t}{y_{ro}^{1t}} \right), i \in I(1), r \in O(1), t = 1, \dots, T \\
\sum_{j=1}^n \lambda_j^{2t} &\left( \frac{x_{ij}^{2t}}{y_{rj}^{2t}} \right) \leq \left( \frac{f_{i2}^t}{y_{ro}^{2t}} \right), i \in I(2), r \in O(2), t = 1, \dots, T \\
\sum_{j=1}^n \lambda_j^{1t} &\left( \frac{x_{ij}^{1t}}{z_{mj}^{1t}} \right) \leq \left( \frac{f_{i1}^t}{z_{mo}^{1t}} \right), i \in I(1), m \in M(1), t = 1, \dots, T \\
\sum_{j=1}^n \lambda_j^{2t} &\left( \frac{x_{mj}^{2t}}{y_{rj}^{2t}} \right) \leq \left( \frac{z_{mo}^{1t}}{y_{ro}^{2t}} \right), m \in M(1), r \in O(2), t = 1, \dots, T \\
\sum_{j=1}^n \lambda_j^{kt} &= p_k, 1 \leq k \leq 2, t = 1, \dots, T \\
\sum_{k=1}^2 p_k &= 1 \\
\lambda_j^{kt} &\geq 0; \forall 1 \leq j \leq n; 1 \leq k \leq 2, t = 1, \dots, T
\end{aligned} \tag{14}$$

Model (14) computes the cost efficiency for the under evaluated DMU in  $t(t = 1, \dots, T)$  time periods. All constraints are similar to model (13). The yawning

gap here is the model (13) is considered only two stage network structure but model (14) is developed for  $t$  ( $t = 1, \dots, T$ ) periods. All constraints are based on DEA-R models as model (13) indicates. As stated before, the overall or total cost

efficiency is obtained as  $C_oE = \frac{\sum_{t=1}^T \left( \sum_{i \in I(1)} f_{i1}^t + \sum_{i \in I(2)} f_{i2}^t \right)}{\sum_{t=1}^T \left( \sum_{i \in I(1)} x_{ij}^{1t} + \sum_{i \in I(2)} x_{ij}^{2t} \right)} \leq 1$ . The existence of con-

straints,  $\sum_{j=1}^n \lambda_j^{kt} = p_k, 1 < k < 2, t = 1, \dots, T, \sum_{k=1}^2 p_k = 1$  enable the decision maker to compute the periodic cost efficiency. For the first stage, the cost efficiency

is calculated as  $C_o^{1t}E = \frac{\sum_{i \in I(1)} f_{i1}^t}{\sum_{i \in I(1)} x_{ij}^{1t}} \leq 1$  and in  $t$  ( $t = 1, \dots, T$ ) time period, the cost

efficiency is  $C_o^1E = \frac{\sum_{t=1}^T \sum_{i \in I(1)} f_{i1}^t}{\sum_{t=1}^T \sum_{i \in I(1)} x_{ij}^{1t}}$ . Clearly, the total overall efficiency is computed as

the weighted average of  $C_o^1E$  and  $C_o^2E$  That is to say, the total cost efficiency for the whole system can be computed as the weighted average of the periodic cost efficiencies in  $t$  ( $t = 1, \dots, T$ ) time period.

**Theorem 1.** *Models (13) and (14) are feasible.*

*Proof.* for the first stage, the constraints  $\sum_{j=1}^n \lambda_j^{2t} = 0$  and  $\sum_{j=1}^n \lambda_j^{1t} = 1$  indicate that  $\lambda_j^{1t} = 0, j \neq o$  and  $\lambda_o^{1t} = 1$  is a feasible solution of the first step and from  $\sum_{j=1}^n \lambda_j^{1t} = 0$  and  $\sum_{j=1}^n \lambda_j^{2t} = 1$  It can be said that  $\lambda_j^{2t} = 0, j \neq o$  and  $\lambda_o^{2t} = 1$  is a feasible solution of the second step. For every ( $1 \leq k \leq 2$ ), assume that  $p_k = \frac{1}{2}$  then from  $\sum_{j=1}^n \lambda_j^{kt} = p_k$  and for each ( $1 \leq k \leq 2$ )  $\lambda_j^{kt} = 0, j \neq o$  and  $\lambda_o^{kt} = \frac{1}{2}$  is a feasible solution of the total network in the whole  $t$  ( $t = 1, \dots, T$ ) It can be concluded that the optimal value does not exceed one and is always greater than zero. Given the final output price vector  $V_j^{1t} = \{v_{rj}^{1t}, r \in O(1)\}$  and  $V_j^{2t} = \{v_{rj}^{2t}, r \in O(1)\}$  for the two stage network structure shown in Fig1. The following DEA-R based model is proposed to measure the revenue efficiency:

$$\begin{aligned}
 & \max \sum_{r \in O(1)} v_r^t f_{r1}^t + \sum_{r \in O(2)} v_r^t f_{r2}^t \\
 & \text{s.t. } \sum_{j=1}^n \lambda_j^{1t} \left( \frac{y_{rj}^{1t}}{x_{io}^{1t}} \right) \geq \left( \frac{f_{r1}^t}{x_{io}^{1t}} \right), i \in I(1), r \in O(1) \\
 & \sum_{j=1}^n \lambda_j^{2t} \left( \frac{y_{rj}^{2t}}{x_{ij}^{2t}} \right) \geq \left( \frac{f_{r2}^t}{x_{io}^{2t}} \right), i \in I(2), r \in O(2) \\
 & \sum_{j=1}^n \lambda_j^{1t} \left( \frac{z_{mj}^{1t}}{x_{ij}^{1t}} \right) \geq \left( \frac{z_{mo}^{1t}}{x_{io}^{1t}} \right), i \in I(1), m \in M(1) \\
 & \sum_{j=1}^n \lambda_j^{2t} \left( \frac{y_{rj}^{2t}}{z_{mj}^{2t}} \right) \geq \left( \frac{f_{r2}^t}{z_{mo}^{2t}} \right), m \in M(1), r \in O(2) \\
 & \sum_{j=1}^n \lambda_j^{kt} = p_k, 1 \leq k \leq 2
 \end{aligned} \tag{15}$$

$$\sum_{k=1}^2 p_k = 1$$

$$\lambda_j^{kt} \geq 0 \quad \forall 1 \leq j \leq n; 1 \leq k \leq 2$$

□

The optimal objective function in model (15) calculates the overall revenue efficiency with regards to periodic efficiency of all DMUs in all periods and stages. As the model (15) implies, the first constraint in model (15) measures the radial increase in the ratio of the value of final outputs to the value of external inputs in the first stage. The second constraint in the model (15) also measures the radial increase in the ratio of the value of the intermediate measures to the external inputs in the second stage. For the second step, the third and fourth constraints are presented similarly. The relation  $V_o^t E = \frac{\sum_{r \in O(1)} y_{rj}^{1t} + \sum_{r \in O(2)} y_{rj}^{2t}}{\sum_{r \in O(1)} f_{r1}^t + \sum_{r \in O(2)} f_{r2}^t} \leq 1$  calculates the revenue efficiency as stated in Fig.1. The development of model (15), for the whole  $t = (t = 1, \dots, T)$  time periods are reformulated as model (16):

$$\begin{aligned} & \max \sum_{t=1}^T \left( \sum_{r \in O(1)} v_r^t f_{r1}^t + \sum_{r \in O(2)} v_r^t f_{r2}^t \right) \\ & s.t. \sum_{j=1}^n \lambda_j^{1t} \left( \frac{y_{rj}^{1t}}{x_{ij}^{1t}} \right) \geq \left( \frac{f_{r1}^t}{x_{io}^1} \right), i \in I(1), r \in O(1), t = 1, \dots, T \\ & \sum_{j=1}^n \lambda_j^{2t} \left( \frac{y_{rj}^{2t}}{x_{ij}^{2t}} \right) \geq \left( \frac{f_{r2}^t}{x_{io}^2} \right), i \in I(2), r \in O(2), t = 1, \dots, T \\ & \sum_{j=1}^n \lambda_j^{1t} \left( \frac{z_{mj}^{1t}}{x_{ij}^{1t}} \right) \geq \left( \frac{z_{mo}^1}{x_{io}^1} \right), i \in I(1), m \in M(1), t = 1, \dots, T \quad (16) \\ & \sum_{j=1}^n \lambda_j^{2t} \left( \frac{y_{rj}^{2t}}{z_{mj}^{1t}} \right) \geq \left( \frac{f_{r2}^t}{z_{mo}^1} \right), m \in M(1), r \in O(2), t = 1, \dots, T \\ & \sum_{j=1}^n \lambda_j^{kt} = p_k, 1 \leq k \leq 2, t = 1, \dots, T \\ & \sum_{k=1}^2 p_k = 1 \\ & \lambda_j^{kt} \geq 0 \quad \forall 1 \leq j \leq n; 1 \leq k \leq 2, t = 1, \dots, T \end{aligned}$$

The optimal solution of the model (16) is the efficient total revenue efficiency with regards to periodic revenue efficiencies. All constraints are similar with a difference of adding the whole  $t(t = 1, \dots, T)$  time periods. The overall network revenue efficiency is identified as:  $V_o E = \frac{\sum_{t=1}^T (\sum_{r \in O(1)} y_{rj}^{1t} + \sum_{r \in O(2)} y_{rj}^{2t})}{\sum_{t=1}^T (\sum_{r \in O(1)} f_{r1}^t + \sum_{r \in O(2)} f_{r2}^t)}$ . Similar to the cost efficiency concept and respect to the parallel structure of the proposed system, the revenue efficiency of the whole network can be written as a weighted average of the periodic revenue efficiencies of each stage of the network.

**Theorem 2.** *Models (15) and (16) are feasible.*

*Proof.* The proof is similar to Theorem 1. In order to obtain the profit efficiency of  $DMU_o$  in presence of first and second stage input and output price vectors, represented as  $C_j^{1t} = \{c_{ij}^{1t}, i \in I(1)\}$ ,  $C_j^{2t} = \{c_{ij}^{2t}, i \in I(2)\}$ ,  $V_j^{1t} = \{v_{rj}^{1t}, r \in O(1)\}$  and  $V_j^{2t} = \{v_{rj}^{2t}, r \in O(2)\}$  respectively, the profit efficiency can be obtained for two stage structure as follows in model (17):

$$\begin{aligned}
& \max \sum_{t=1}^T \left( \sum_{r \in O(1)} v_r^t f_{r1}^t + \sum_{r \in O(2)} v_r^t f_{r2}^t \right) - \left( \sum_{i \in I(1)} c_i^t f_{i1}^t + \sum_{i \in I(2)} c_i^t f_{i2}^t \right) \\
& \text{s.t. } \sum_{j=1}^n \lambda_j^{1t} \left( \frac{x_{ij}^{1t}}{y_{rj}^{1t}} \right) \leq \left( \frac{f_{i1}^t}{y_{ro}^{1t}} \right), i \in I(1), r \in O(1), \\
& \sum_{j=1}^n \lambda_j^{2t} \left( \frac{x_{ij}^{2t}}{y_{rj}^{2t}} \right) \leq \left( \frac{f_{i2}^t}{y_{ro}^{2t}} \right), i \in I(2), r \in O(2), \\
& \sum_{j=1}^n \lambda_j^{1t} \left( \frac{x_{ij}^{1t}}{z_{mj}^{1t}} \right) \leq \left( \frac{f_{i1}^t}{z_{mo}^{1t}} \right), i \in I(1), m \in M(1), \\
& \sum_{j=1}^n \lambda_j^{2t} \left( \frac{z_{mj}^{1t}}{y_{rj}^{2t}} \right) \leq \left( \frac{z_{mo}^{1t}}{y_{ro}^{2t}} \right), m \in M(1), r \in O(2) \\
& \sum_{j=1}^n \lambda_j^{1t} \left( \frac{y_{rj}^{1t}}{x_{ij}^{1t}} \right) \geq \left( \frac{f_{r1}^t}{x_{io}^{1t}} \right), i \in I(1), r \in O(1) \\
& \sum_{j=1}^n \lambda_j^{2t} \left( \frac{y_{rj}^{2t}}{x_{ij}^{2t}} \right) \geq \left( \frac{f_{r2}^t}{x_{io}^{2t}} \right), i \in I(2), r \in O(2) \\
& \sum_{j=1}^n \lambda_j^{1t} \left( \frac{z_{mj}^{1t}}{x_{ij}^{1t}} \right) \geq \left( \frac{z_{mo}^{1t}}{x_{io}^{1t}} \right), i \in I(1), m \in M(1) \\
& \sum_{j=1}^n \lambda_j^{2t} \left( \frac{y_{rj}^{2t}}{z_{mj}^{1t}} \right) \geq \left( \frac{f_{r2}^t}{z_{mo}^{1t}} \right), m \in M(1), r \in O(2) \\
& \sum_{j=1}^n \lambda_j^{kt} = p_k, 1 \leq k \leq 2 \\
& \sum_{k=1}^2 p_k = 1 \\
& \lambda_j^{kt} \geq 0 \forall 1 \leq j \leq n; 1 \leq k \leq 2
\end{aligned} \tag{17}$$

Employing the optimal solution of model (17) the relation  $P_o^t E = \frac{\sum_{r \in O(1)} y_{rj}^{1t} + \sum_{r \in O(2)} y_{rj}^{2t} - (\sum_{i \in I(1)} x_{ij}^{1t} + \sum_{i \in I(2)} x_{ij}^{2t})}{\sum_{r \in O(1)} f_{r1}^t + \sum_{r \in O(2)} f_{r2}^t - (\sum_{i \in I(1)} f_{i1}^t + \sum_{i \in I(2)} f_{i2}^t)}$  indicated the profit efficiency for the two stage network structures. Based on the axioms of DEA-R model, extension of the model in  $t(t = 1, \dots, T)$  time periods model (18) computes the total profit

efficiency for multi period network structure. The model has the following format.

$$\begin{aligned}
& \max \sum_{t=1}^T \left( \sum_{r \in O(1)} v_r^t f_{r1}^t + \sum_{r \in O(2)} v_r^t f_{r2}^t \right) - \sum_{t=1}^T \left( \sum_{i \in I(1)} c_i^t f_{i1}^t + \sum_{i \in I(2)} c_i^t f_{i2}^t \right) \\
& \text{s.t. } \sum_{j=1}^n \lambda_j^{1t} \left( \frac{x_{ij}^{1t}}{y_{rj}^{1t}} \right) \leq \left( \frac{f_{i1}^t}{y_{rj}^{1t}} \right), i \in I(1), r \in O(1), t = 1, \dots, T \\
& \sum_{j=1}^n \lambda_j^{2t} \left( \frac{x_{ij}^{2t}}{y_{rj}^{2t}} \right) \leq \left( \frac{f_{i2}^t}{y_{rj}^{2t}} \right), i \in I(2), r \in O(2), t = 1, \dots, T \\
& \sum_{j=1}^n \lambda_j^{1t} \left( \frac{x_{ij}^{1t}}{z_{mj}^{1t}} \right) \leq \left( \frac{f_{i1}^t}{z_{mj}^{1t}} \right), i \in I(1), m \in M(1), t = 1, \dots, T \\
& \sum_{j=1}^n \lambda_j^{2t} \left( \frac{z_{mj}^{1t}}{y_{rj}^{2t}} \right) \leq \left( \frac{z_{mj}^{1t}}{y_{rj}^{2t}} \right), m \in M(1), r \in O(2), t = 1, \dots, T \tag{18} \\
& \sum_{j=1}^n \lambda_j^{1t} \left( \frac{y_{rj}^{1t}}{x_{ij}^{1t}} \right) \geq \left( \frac{f_{r1}^t}{x_{io}^1} \right), i \in I(1), r \in O(1), t = 1, \dots, T \\
& \sum_{j=1}^n \lambda_j^{2t} \left( \frac{y_{rj}^{2t}}{x_{ij}^{2t}} \right) \geq \left( \frac{f_{r2}^t}{x_{io}^2} \right), i \in I(2), r \in O(2), t = 1, \dots, T \\
& \sum_{j=1}^n \lambda_j^{1t} \left( \frac{z_{mj}^{1t}}{x_{ij}^{1t}} \right) \geq \left( \frac{z_{mj}^{1t}}{x_{io}^1} \right), i \in I(1), m \in M(1), t = 1, \dots, T \\
& \sum_{j=1}^n \lambda_j^{2t} \left( \frac{y_{rj}^{2t}}{z_{mj}^{1t}} \right) \geq \left( \frac{f_{r2}^t}{z_{mj}^{1t}} \right), m \in M(1), r \in O(2), t = 1, \dots, T \\
& \sum_{j=1}^n \lambda_j^{kt} = p_k, 1 \leq k \leq 2, T = 1, \dots, T \\
& \sum_{k=1}^2 p_k = 1 \\
& \lambda_j^{kt} \geq 0 \forall 1 \leq j \leq n; 1 \leq k \leq 2, t = 1, \dots, T \tag{19}
\end{aligned}$$

As mentioned before, the constraints are similar to model (17). Employing the optimal solution of model (18), the total profit efficiency is at hand applying the following relationship:

$$P_o E = \frac{\sum_{t=1}^T \left( \sum_{r \in O(1)} y_{rj}^{1t} + \sum_{r \in O(2)} y_{rj}^{2t} - \left( \sum_{i \in I(1)} x_{ij}^{1t} + \sum_{i \in I(2)} x_{ij}^{2t} \right) \right)}{\sum_{t=1}^T \left( \sum_{r \in O(1)} f_{r1}^t + \sum_{r \in O(2)} f_{r2}^t - \left( \sum_{i \in I(1)} f_{i1}^t + \sum_{i \in I(2)} f_{i2}^t \right) \right)} \leq 1$$

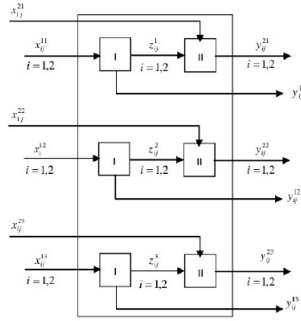
Similar to cost and revenue efficiency, the profit efficiency of the entire network can be written as a weighted average of the periodic profit efficiencies of each stage. As before, the proposed models are always feasible. The following theorem shows this fact.  $\square$

**Theorem 3.** *Models (17) and (18) are feasible.*

*Proof.* The proof is similar to Theorem 1 □

#### 4. Numerical example

To show the applicability and merits of the proposed method and meanwhile to compare it with the Models (13), (14), (15), (16), (17) and (18), Data set of a real case of ten decision making units are taken from [24]. Each decision making unit has a two-stage structure and the each stage has observed in three time periods. Figure 2 presents the structure of the multi period system. Each decision making unit  $DMU_j (j = 1, \dots, 10)$ , in the first stage consume two external inputs  $x_{1j}^{1t}$  and  $x_{2j}^{1t}$  to produce two intermediate products  $z_{1j}^{1t}$  and  $z_{2j}^{1t}$  and a final output  $y_{1j}^{1t}$ . The second stage fed up with three inputs which consists of two intermediate products from the first stage and one external input  $x_{1j}^{2t}$ . The second stage final outputs are characterized as  $y_{1j}^{2t}$  and  $y_{2j}^{2t}$ . The superscript  $t (t = 1, \dots, 3)$  indicated three time periods. The statistics for the three time periods are depicted



**Figure 2.** Multi-period network production system with  $P=2, T=3$

in Table 1. The input and output cost vectors for three periods 1, 2 and 3 as

**Table 1.** Mean and standard deviation of the data set in three time periods.

first time	A	B	C	D	E	F	G	H	I	J
mean	7.2500	6.6250	8.0000	8.7500	7.3750	7.7500	7.3750	6.8750	5.5000	7.7500
std.Deviation	4.83292	3.37797	5.12696	5.33854	3.73927	4.74342	3.37797	2.85044	31.6228	4.33425
second time	A	B	C	D	E	F	G	H	I	J
mean	8.3375	9.0000	11.0375	12.0125	9.6500	9.6375	9.4375	8.91255	7.5125	10.0500
std.Deviation	4.97822	4.79434	8.03473	7.82221	5.13893	5.65229	4.47786	3.97076	4.61006	5.75872
Third time	A	B	C	D	E	F	G	H	I	J
mean	9.9862	13.5000	16.5562	20.3588	14.4562	12.5938	12.5125	12.4375	10.3562	12.4857
std.Deviation	5.39055	7.191251	1.20521	1.21523	3.7.24527	6.56552	5.78111	4.58661	60.03806	7.02197

$$(c_1^1, c_2^1, c_3^1) = (2, 1, 2), (v_1^1, v_2^1, v_3^1) = (5, 2, 4), (c_1^2, c_2^2, c_3^2) = (2, 1, 2), (v_1^2, v_2^2, v_3^2) =$$

$(6, 1, 3)$ ,  $(c_1^3, c_2^3, c_3^3) = (3, 2, 3)$ ,  $(v_1^3, v_2^3, v_3^3) = (9, 4, 5)$ , are stated respectively. Given the cost vectors, the proposed model(13), (14) are implemented. The results are represented in Tables 2, 3 and 4. The efficiency of overall cost, revenue and profit of the

Table 2. Multi-period cost efficiency of the overall network after three time periods

DMU	A	B	C	D	E	F	G	H	I	J
$C_oE$	0.862806	0.657050	0.833422	0.785117	0.815294	0.623722	0.495482	0.886518	0.800067	0.911442

system and each of the stages of the network have been investigated in an arbitrary time period and after 3 time periods by means of models (13), (14), (15),(16),(17) and (18). Table 2 shows the results of solving model (14). The columns of this table show the cost efficiency of each of the decision-making units of the entire network after three time periods. According to this table, unit J has the highest efficiency of cost and unit G has the lowest overall efficiency of cost. Table 3 shows the results

Table 3. Cost efficiency of the overall network in each time period

DMU	$C_o^1E$	$C_o^2E$	$C_o^3E$
A	0.875000	0.833774	0.871279
B	0.547404	0.593797	0.722402
C	0.830000	0.841772	0.830991
D	0.855856	0.850767	0.731974
E	0.966706	0.864159	0.746544
F	0.629630	0.641373	0.613701
G	0.612696	0.505506	0.450481
H	0.764138	0.740219	1. 000000
I	0.831535	0.807339	0.786744
J	0.956263	0.954983	0.878063

of solving model (13). The columns of this table show each of the decision-making units with the cost efficiency of the entire network in each time period. According to Table 3, Unit H is cost efficient in the third period, but because it is inefficient in the first and second periods, it is also inefficient after 3 periods. Table 4 depicts the cost efficiency of each stage of the network and the entire network after three time periods. According to Table 4, units C and F are cost efficient in the second stage, but in the entire network, these units are inefficient, and this is due to the first stage. Also According to Tables 2 and 4, it can be seen that the overall efficiency results are the same. Using this method, the contribution of each stage and time period in the evaluation of the efficiency of the total cost is determined. Running the revenue efficiency models (15),(16) on the data set of Table 1 with the given price vectors, the resulted are showed in Tables 5,6,7.

Table 5 shows the results of solving model (16). The columns of this table show each

Table 4. Cost efficiency of each of the network stages and the entire network after three time periods

DMU	$C_o^I E$	$C_o^{II} E$	$C_o^{III} E$
A	0.697770	0.979678	0.862809
B	0.737227	0.590429	0.657050
C	0.678115	1.000000	0.833422
D	0.819084	0.756319	0.785117
E	0.994560	0.637281	0.815294
F	0.374661	1.000000	0.623722
G	0.433428	0.616649	0.495482
H	0.906784	0.867119	0.886518
I	1.000000	0.666717	0.800067
J	0.904735	0.922961	0.911442

Table 5. Multi-period revenue efficiency of the overall network after three time periods

DMU	A	B	C	D	E	F	G	H	I	J
$V_o E$	0.858404	0.750996	0.831591	0.770771	0.781308	0.593123	0.486221	0.673130	0.764032	0.800225

of the decision-making units with the revenue efficiency of the entire network after three time periods. According to this table, unit A has the highest revenue efficiency and unit G has the lowest overall revenue efficiency. Table 6 shows the results of

Table 6. Revenue efficiency in each time period and efficiency of the overall multi-period system

DMU	$V_o^1 E$	$V_o^2 E$	$V_o^3 E$
A	0.895833	0.816876	0.862101
B	0.618229	0.709899	0.820392
C	0.782609	0.728358	0.889530
D	0.781484	0.878620	0.7335515
E	0.802985	0.825130	0.760455
F	0.631336	0.547990	0.601156
G	0.490415	0.434950	0.510539
H	0.528306	0.502894	0.877096
I	0.799311	0.834050	0.726195
J	0.858609	0.822437	0.770057

solving model (15). The columns of this table show each of the decision-making units with the revenue efficiency of the entire network in each time periods. Table 7 shows the revenue efficiency of each stage of the network and the entire network. According to table 7, units C and F are efficient in the second stage, but due to the inefficiency in the first stage, they are inefficient in the entire network. Units D , E and I are also



Table 7. Revenue efficiency in each time period and efficiency of the overall multi-period system

DMU	$V_o^I E$	$V_o^{II} E$	$V_o^{III} E$
A	0.746221	0.981823	0.858404
B	0.840715	0.665616	0.750996
C	0.627390	1.000000	0.831591
D	1.000000	0.616565	0.770771
E	1.000000	0.672223	0.781308
F	0.408843	1.000000	0.593123
G	0.378409	0.716621	0.486221
H	0.590460	0.747899	0.673130
I	1.000000	0.623456	0.764032
J	0.742487	0.904511	0.800225

efficient in the first stage and inefficient in the second stage, so they are inefficient in the entire network. Similarly, According to tables 5, 7 it can be seen that the overall efficiency results are the same. The proposed method determines the contribution of each stage and time period in the evaluation of the efficiency of the total revenue. Applying proposed profit efficiency models (17),(18) on the data set of Table1, the results are analyzed in Tables 8,9 and 10. Table 8 shows the results of solving model

Table 8. Multi-period profit efficiency of the overall network after three time periods

DMU	A	B	C	D	E	F	G	H	I	J
$P_o E$	0.792614	0.668017	0.778264	0.7362561	0.711437	0.452854	0.360438	0.606278	0.701385	0.762758

(18) and indicates the profit efficiency of the entire network system after three time periods. According to this table, unit C has the highest profit efficiency and unit G has the lowest overall profit efficiency. Table 9 shows the results of solving model(

Table 9. Profit efficiency in each time period and efficiency of the overall multi-period system

DMU	$P_o^1 E$	$P_o^2 E$	$P_o^3 E$
A	0.853659	0.746388	0.785981
B	0.506385	0.600245	0.765761
C	0.720143	0.658257	0.845379
D	0.796609	0.960549	0.669773
E	0.762892	0.753466	0.684084
F	0.509238	0.417511	0.447967
G	0.377465	0.313253	0.378224
H	0.413212	0.371557	1.000000
I	0.744897	0.776036	0.659697
J	0.835629	0.797148	0.722066

17) and depicts the profit efficiency in the desired time period. According to table 9, unit H is profit efficient in the third period, but because it is inefficient in the first and second periods, it is also profit inefficient after 3 periods. Table 10 shows

Table 10. Profit efficiency of each network stage and the entire network after three time periods

DMU	$P_o^I E$	$P_o^{II} E$	$P_o^{III} E$
A	0.655427	0.972134	0.792614
B	0.809598	0.537242	0.668017
C	0.514211	1.000000	0.778264
D	1.000000	0.562372	0.736256
E	1.000000	0.597308	0.711437
F	0.254219	1.000000	0.452854
G	0.237331	0.654082	0.360438
H	0.453791	0.761912	0.606278
I	1.000000	0.535303	0.701385
J	0.691714	0.893551	0.762758

the profit efficiency of each stage of the network and the entire network. According to Table 10, units C and F are profit efficient in the second stage, but they are inefficient in the whole network. Units D , E and I are profit efficient in the first stage, but they are inefficient in the entire network. According to Table 8 and 10, it can be seen that the results of the overall profit efficiency are the same.

So, considering these results, we can say that inefficiency in a period or a stage affects the efficiency of the entire network system. The proposed methods for calculating the efficiency of cost, revenue and profit enable us to determine the contribution of each of the stages and periods of time in the amount of achieved efficiency.

## 5. Conclusion

Ratio-based data is an interesting and challenging issue to be used in the Data Envelopment Analysis (DEA) context, particularly, in almost all financial institutions having access to numerical ratios where they are able to compare their units (such as Quick ratio = assets to liabilities) .DEA society believes that introducing ratio-based data in DEA by Despic et al. [3] has been on time. On the other hand, many researchers would like to know how the cost, revenue, and profit efficiency of a set of *DMUs* could be measured in a specific period of time. When the time span for efficiency measurement involves multiple periods, the overall efficiency can be obtained as an average through all periods under consideration . However, one of the drawbacks of this method is that it ignores the operations of individual periods. To overcome to this problem , a DEA-R model is developed to consider the total efficiency and also periodic efficiencies simultaneously. Then the conventional DEA-R has been modified to incorporate every individual period in overall efficiency mea-

surement for distinguishing cost, revenue and profit efficient and inefficient DMUs. We further showed that the overall cost, revenue, and profit efficiency in a time period  $t$  ( $t = 1, \dots, T$ ) are the weighted average of periodic cost, revenue, and profit efficiency. Finally, the superiority of the proposed model showed by applying it to ten decision-making units. Since the data are not known precisely then it will be suitable to investigate this problem when data are given as fuzzy numbers or uncertain data using uncertainty analysis which is postponed as a future work.

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