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Research Article

Sombor index of product of graphs

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Abstract: Recently a new vertex-degree based molecular structure descriptor was defined as Sombor index. For a simple graph G, the Sombor index of G, denoted by $SO(G)$, is defined as $\sum_{uv\in E(G)} \sqrt{d_u^2 + d_v^2}$, where d_v is the degree of v. In this paper we study the Sombor index of many kinds of product of graphs, such as join of graphs, Cartesian product of graphs, tensor product of graphs, and lexicographic product of graphs. We obtain some formulas for the Sombor index of these product of graphs.

Keywords: sombor index of graphs, product of graphs.

AMS Subject classification: 05C07, 05C09

1. Introduction

Throughout the paper, the graphs are simple. In other words, they are finite and undirected, without loops and multiple edges. Let $G = (V(G), E(G))$ be a simple graph. By *order* of G we mean the number of vertices of G. The *size* of G is the number of edges of G. By $e = uv$ we mean the edge e between u and v. For a vertex $v \in V(G)$, the *degree* of v is the number of edges incident with v and is denoted by $\deg_G(v)$ (sometimes we use $\deg v$ instead of $\deg_G(v)$ when the graph G determine from the text) or $deg(v, G)$. A *pendant* vertex is a vertex with degree one and a pendant edge is an edge such that one of its end points is pendant vertex. A $k-$*regular* graph is a graph such that every vertex of that has degree k. By $\delta(G)$ and $\Delta(G)$ we mean the minimum vertex degree and the maximum vertex degree of vertices of G, respectively. The *complement* of a graph G is denoted by $\overline{G}$. An *independent set* S in G is a subset of vertices of G such that the vertices of S are not adjacent. The *edgeless graph* (or *empty graph*), the *complete graph*, the *cycle*, and the *path* of order n, are denoted by $\overline{K_n}$, K_n, C_n and P_n, respectively. Let t and $n_1, \ldots, n_t$ be some positive integers. By $K_{n_1,\ldots,n_t}$ we mean the *complete multipartite graph* with parts size $n_1, \ldots, n_t$. In particular, the *complete bipartite graph* with part sizes m and n is denoted by $K_{m,n}$. The *star* of order n, denoted by S_n, is the complete bipartite graph $K_{1,n-1}$.

In chemical graph theory there are many topological indices. Recently, a new index, Sombor index, has been defined by Ivan Gutman in [5]. For a graph G, the Sombor index of G, denoted by $SO(G)$, is defined as

$$SO(G) = \sum_{uv \in E(G)} \sqrt{d_u^2 + d_v^2},$$

where d_v is the degree of v. For example the Sombor index of the star S_n is $(n-1)\sqrt{(n-1)^2+1}$. There are many papers related to properties of Sombor index, for instance see [1–12, 15, 18] and the references therein.

In graph theory, they are many kinds of product on graphs. Now we recall some important of them. Let G and H be two disjoint graphs. The *disjoint union* of G and H, denoted by $G \cup H$, is the graph with the vertex set $V(G) \cup V(H)$ and the edge set $E(G) \cup E(H)$. The graph rG denotes the disjoint union of r copies of G. The *join* of G and H that is denoted by $G \vee H$ is the graph with vertex set $V(G) \cup V(H)$ and the edge set $E(G) \cup E(H) \cup \{uv : u \in V(G) \,\text{and}\, v \in V(H)\}$.

The *Cartesian product* of the disjoint graphs G and H, denoted by $G \Box H$, is the graph with vertex set $V(G) \times V(H)$ and two distinct vertices (u, v) and (u', v') are adjacent in $G \Box H$ if and only if $u = u'$ and v is adjacent to v' in H or $v = v'$ and u is adjacent to u' in G. The *lexicographic product* of two disjoint graphs G and H, denoted by $G[H]$ is the graph with vertex set $V(G) \times V(H)$ and two vertices (u, v) and (u', v') are adjacent in $G[H]$ if and only if either u is adjacent with u' in G or $u = u'$ and v is adjacent with v' in H. The *direct product* (or *tensor product*) of G and H denoted by $G \times H$, is the graph with the vertex set $V(G) \times V(H)$, and two distinct vertices (u, u') and (v, v') are adjacent in $G \times H$ if and only if u is adjacent to v in G and u' is adjacent to v' in H.

There are a few papers related to the Sombor index of product of graphs. The most parts of those papers are related to computation the Sombor index of some special graphs, see [13, 14, 16, 17]. In this paper we study the Sombor index of the above product of graphs. More precisely, we find some relations between $SO(G \star H)$ and $SO(G)$ and $SO(H)$, where $\star$ denotes an operation product on graphs G and H.

2. Join of graphs

We recall that a *spanning* subgraph of a graph G is a subgraph of G with same vertex set as the vertex set of G. Let G be a graph. By $G = (H_1, \ldots, H_k)$ we mean that $H_1, \ldots, H_k$ are some spanning subgraphs of G such that the edge sets of $H_1, \ldots, H_k$ are disjoint ($E(H_i) \cap E(H_j) = \emptyset$ for every $i \neq j$) and $E(G) = E(H_1) \cup \cdots \cup E(H_k)$. In this case we say that $H_1, \ldots, H_k$ are a edge-disjoint partition of G. For example $C_6 = (3K_2, 3K_2)$. In [11], the following result has been proved.

Theorem 1. [11] *Let G be a connected graph. Assume that $G = (H_1, \ldots, H_k)$. Then*

$$SO(G) \geq SO(H_1) + \cdots + SO(H_k).$$

Moreover, the equality holds if and only if for some $j \in \{1, \ldots, k\}$, $H_j = G$ and the other subgraphs are empty graphs.

Now we are is a position to prove our results. We begin by the clear following remark.

Remark 1. Let G_1 and G_2 be two disjoint graphs. Then $SO(G_1 \cup G_2) = SO(G_1) + SO(G_2)$.

Using Remark 1 and Theorem 1 one can prove a general version of Theorem 1.

Theorem 2. *Let G be a graph. Assume that $G = (H_1, \ldots, H_k)$. Then*

$$SO(G) \geq SO(H_1) + \cdots + SO(H_k).$$

In the next theorem we find a relation for the Sombor index of join of graphs.

Theorem 3. *Let G_1 and G_2 be two disjoint graphs with order n_1 and n_2, respectively. Then*

$$SO(G_1 \vee G_2) \geq SO(G_1) + SO(G_2) + SO(K_{n_1,n_2}) \tag{2.1}$$

and the equality holds if and only if $G_1 = \overline{K_{n_1}}$ and $G_2 = \overline{K_{n_2}}$.

Proof. Let $H_1 = G_1 \cup \overline{K_{n_2}}$ and $H_2 = G_2 \cup \overline{K_{n_1}}$ and $H_3 = K_{n_1,n_2}$. Therefore $G_1 \vee G_2 = (H_1, H_2, H_3)$. Since $G_1 \vee G_2$ is a connected graph, by Theorem 1 we find that

$$SO(G_1 \vee G_2) \geq SO(H_1) + SO(H_2) + SO(H_3) \tag{2.2}$$

On the other hand by Remark 3, $SO(H_1) = SO(G_1)$, $SO(H_2) = SO(G_2)$ and $SO(H_3) = SO(K_{n_1,n_2}) = n_1 n_2 \sqrt{n_1^2 + n_2^2}$. Hence inequality (2.1) follows.
Now we check the equality of (2.1). By Theorem 1 in (2.2) the equality equality holds if and only if $H_1 = G_1 \vee G_2$, $H_2 = \overline{K_{n_1+n_2}}$, $H_3 = \overline{K_{n_1+n_2}}$ or $H_2 = G_1 \vee G_2$, $H_1 = \overline{K_{n_1+n_2}}$, $H_3 = \overline{K_{n_1+n_2}}$ or $H_3 = G_1 \vee G_2$, $H_1 = \overline{K_{n_1+n_2}}$, $H_2 = \overline{K_{n_1+n_2}}$. It is easy to check that only the last case can be happen. In other words, the equality holds if and only if $H_3 = G_1 \vee G_2$, $H_1 = \overline{K_{n_1+n_2}}$, $H_2 = \overline{K_{n_1+n_2}}$. Hence the equality holds if and only if $G_1 = \overline{K_{n_1}}$ and $G_2 = \overline{K_{n_2}}$ (checking the converse of equality is easy). This completes the proof. □

3. Cartesian product and lexicographic product of graphs

In this section we study the Sombor index of Cartesian product and lexicographic product of graphs. At first we consider the Cartesian product of connected graphs.

Theorem 4. *Let G_1 and G_2 be two disjoint connected graphs with order n_1 and n_2, respectively. Then*

$$SO(G_1 \Box G_2) \geq n_2 SO(G_1) + n_1 SO(G_2) \tag{3.1}$$

and the equality holds if and only if $G_1 = K_1$ or $G_2 = K_1$.

Proof. Let $H_1 = G_1 \cup \overline{K_{n_1 n_2 - n_1}}$ and $H_2 = G_2 \cup \overline{K_{n_1 n_2 - n_2}}$. One can see that $G_1 \Box G_2 = (\underbrace{H_1, \ldots, H_1}_{n_2}, \underbrace{H_2, \ldots, H_2}_{n_1})$. Since $G_1 \Box G_2$ is connected, by Theorem 1,

$$SO(G_1 \Box G_2) \geq n_2 SO(H_1) + n_1 SO(H_2) \tag{3.2}$$

On the other hand by Remark 3, $SO(H_1) = SO(G_1)$ and $SO(H_2) = SO(G_2)$. Hence inequality (3.1) follows.
Now we check the equality of (3.1). By Theorem 1 in (3.2) the equality holds if and only if $H_1 = G_1 \Box G_2$, $n_2 = 1$ and $H_2 = \overline{K_{n_1 n_2}}$ or $H_2 = G_1 \Box G_2$, $n_1 = 1$ and $H_1 = \overline{K_{n_1 n_2}}$. The first case shows that $G_2 = K_1$ and the second case shows that $G_1 = K_1$. Therefore the equality holds if and only if $G_1 = K_1$ or $G_2 = K_1$ (checking the converse of equality is easy). This completes the proof. □

Now we can generalize Theorem 4 for all graphs.

Theorem 5. *Let G_1 and G_2 be two disjoint graphs with order n_1 and n_2, respectively. Then*

$$SO(G_1 \Box G_2) \geq n_2 SO(G_1) + n_1 SO(G_2) \tag{3.3}$$

and the equality holds if and only if $G_1 = \overline{K_{n_1}}$ or $G_2 = \overline{K_{n_2}}$.

Proof. Assume that $A_1, \ldots, A_k$ are the connected components of G_1 and $B_1, \ldots, B_s$ are the connected components of G_2. Thus $G_1 = A_1 \cup \cdots \cup A_k$ and $G_2 = B_1 \cup \cdots \cup B_s$. One can see that

$$G_1 \Box G_2 = \cup_{i=1}^{k} \cup_{j=1}^{s} A_i \Box B_j.$$

Thus by Remark 1

$$SO(G_1 \Box G_2) = \sum_{i=1}^{k} \sum_{j=1}^{s} SO(A_i \Box B_j). \tag{3.4}$$

Suppose that n'_i and m'_j are the order of A_i and B_j, respectively, for $i = 1, \dots, k$ and $j = 1, \dots, s$. Thus $n_1 = n'_1 + \cdots + n'_k$ and $n_2 = m'_1 + \cdots + m'_s$. Using Theorem 4 we find that

$$\begin{aligned}\sum_{i=1}^{k}\sum_{j=1}^{s} SO(A_i \Box B_j) &\geq \sum_{i=1}^{k}\sum_{j=1}^{s}(m'_j SO(A_i) + n'_i SO(B_j)) \\ &= \sum_{j=1}^{s} m'_j \sum_{i=1}^{k} SO(A_i) + \sum_{i=1}^{k} n'_i \sum_{j=1}^{s} SO(B_j) \end{aligned} \tag{3.5}$$

and the equality holds if and only if for $i = 1, \dots, k$ and $j = 1, \dots, s$, $A_i = K_1$ or $B_j = K_1$. Thus the equality holds if and only if $A_1 = A_2 = \cdots = A_k = K_1$ or $B_1 = B_2 = \cdots = B_s = K_1$. That is $G_1 = \overline{K}_{n_1}$ or $G_2 = \overline{K}_{n_2}$. On the other hand, by Remark 1

$$\sum_{j=1}^{s} m'_j \sum_{i=1}^{k} SO(A_i) + \sum_{i=1}^{k} n'_i \sum_{j=1}^{s} SO(B_j) = n_2 SO(G_1) + n_1 SO(G_2). \tag{3.6}$$

Now by combining (3.4), (3.5) and (3.6) we conclude that

$$SO(G_1 \Box G_2) \geq n_2 SO(G_1) + n_1 SO(G_2)$$

and the equality holds if and only if $G_1 = \overline{K_{n_1}}$ or $G_2 = \overline{K_{n_2}}$. The proof is complete. □

In continue we investigate the Sombor index of the lexicographic product of graphs. One can check the following remark.

Remark 2. Let G and H be two graphs. Then $G[H]$ is connected if and only if G is connected.

Theorem 6. *Let G_1 and G_2 be two disjoint graphs with order n_1 and n_2, respectively. Suppose that G_1 is connected. Then*

$$SO(G_1[G_2]) \geq n_2 SO(G_1) + n_1 SO(G_2) \tag{3.7}$$

and the equality holds if and only if $G_1 = K_1$ or $G_2 = K_1$.

Proof. Let $H_1 = G_1 \cup \overline{K_{n_1 n_2 - n_1}}$ and $H_2 = G_2 \cup \overline{K_{n_1 n_2 - n_2}}$. One can see that $G_1[G_2] = (\underbrace{H_1, \dots, H_1}_{n_2}, \underbrace{H_2, \dots, H_2}_{n_1})$. Since (by Remark 2) $G_1[G_2]$ is connected, by applying Theorem 1 we find that

$$SO(G_1[G_2]) \geq n_2 SO(H_1) + n_1 SO(H_2) \tag{3.8}$$

On the other hand by Remark 3, $SO(H_1) = SO(G_1)$ and $SO(H_2) = SO(G_2)$. Therefore inequality (3.7) follows.
Now we check the equality of (3.7). By Theorem 1 in (3.8) the equality holds if and only if $H_1 = G_1[G_2]$, $n_2 = 1$ and $H_2 = \overline{K_{n_1 n_2}}$ or $H_2 = G_1[G_2]$, $n_1 = 1$ and $H_1 = \overline{K_{n_1 n_2}}$. The first case shows that $G_2 = K_1$ and the second case shows that $G_1 = K_1$. So the equality holds if and only if $G_1 = K_1$ or $G_2 = K_1$ (checking the converse of equality is easy). This completes the proof. □

We end this section by a generalization of Theorem 6 for all graphs.

Theorem 7. *Let G_1 and G_2 be two disjoint graphs with order n_1 and n_2, respectively. Then*

$$SO(G_1[G_2]) \geq n_2 SO(G_1) + n_1 SO(G_2) \tag{3.9}$$

and the equality holds if and only if $G_1 = \overline{K_{n_1}}$ or $G_2 = K_1$.

Proof. Suppose that $A_1, \dots, A_k$ are the connected components of G_1. Thus $G_1 = A_1 \cup \dots \cup A_k$. One can check that

$$G_1[G_2] = A_1[G_2] \cup A_2[G_2] \cup \dots \cup A_k[G_2].$$

Thus by Remark 1

$$SO(G_1[G_2]) = \sum_{i=1}^{k} SO(A_i[G_2]). \tag{3.10}$$

Suppose that for $i = 1, \dots, k$, n'_i is the order of A_i. Thus $n_1 = n'_1 + \dots + n'_k$. By Theorem 6 we obtain that

$$\sum_{i=1}^{k} SO(A_i[G_2]) \geq \sum_{i=1}^{k} (n_2 SO(A_i) + n'_i SO(G_2)) = n_2 \sum_{i=1}^{k} SO(A_i) + \sum_{i=1}^{k} n'_i SO(G_2). \tag{3.11}$$

and the equality holds if and only if for $i = 1, \dots, k$, $A_i = K_1$ or $G_2 = K_1$. Thus the equality holds if and only if $A_1 = A_2 = \dots = A_k = K_1$ or $G_2 = K_1$. That is $G_1 = \overline{K}_{n_1}$ or $G_2 = K_1$. On the other hand, by Remark 1

$$n_2 \sum_{i=1}^{k} SO(A_i) + \sum_{i=1}^{k} n'_i SO(G_2) = n_2 SO(G_1) + n_1 SO(G_2) \tag{3.12}$$

Now by considering (3.10), (3.11) and (3.12) we conclude that

$$SO(G_1[G_2]) \geq n_2 SO(G_1) + n_1 SO(G_2)$$

and the equality holds if and only if $G_1 = \overline{K_{n_1}}$ or $G_2 = K_1$. The proof is complete. □

4. Direct product of graphs

In this section we study the Sombor index of direct product of graphs. We note that the direct product of graphs are also called the tensor product of graphs and the Kronecker product of graphs. The Sombor index of direct product of graphs is more complicated than the previous product of graphs. We start with the following remark.

Remark 3. Let G and H be two graphs. Suppose that G is bipartite with parts X and Y. One can see that $X \times V(H)$ and $Y \times V(H)$ are two independent sets in $G \times H$. Therefore $G \times H$ is bipartite.

We note that if G and H are connected, then $G \times H$ is not essentially connected. Now we prove one of the main results of this section.

Theorem 8. *Let G_1 and G_2 be two disjoint graphs with size m_1 and m_2, respectively. Suppose that G_1 is bipartite. Then*

$$SO(G_1 \times G_2) \geq 2m_2 SO(G_1). \tag{4.1}$$

Moreover, if $G_1 \times G_2$ is connected, then $SO(G_1 \times G_2) > 2m_2 SO(G_1)$.

Proof. Let X and Y be the parts of the bipartite graph G_1. Assume that $e = ab$ is an edge of G_2. One can see that the induced subgraph on $X \times \{a\} \cup Y \times \{b\}$ in $G_1 \times G_2$ is isomorphic to G_1. In addition, $X \times \{b\} \cup Y \times \{a\}$ in $G_1 \times G_2$ is isomorphic to G_1. Thus for every edge of G_2, we have two edge-disjoint copies of G_1. By considering these kinds of subgraphs, it is not hard to see that $G_1 \times G_2$ can be partitioned to $2m_2$ edge-disjoint copies of G_1. Thus by Theorem 1, $SO(G_1 \times G_2) \geq 2m_2 SO(G_1)$. If $G_1 \times G_2$ is connected, then by Theorem 1 the equality does not hold. □

In the next result we investigate the Sombor index of direct product of two bipartite graphs.

Theorem 9. *Let G_1 and G_2 be two disjoint bipartite graphs with size m_1 and m_2, respectively. Then*

$$SO(G_1 \times G_2) \geq \max\{2m_2 SO(G_1), 2m_1 SO(G_2), m_2 SO(G_1) + m_1 SO(G_2)\}. \tag{4.2}$$

Proof. Let X_1 and Y_1 be the parts of the bipartite graph G_1 and, X_2 and Y_2 be the parts of the bipartite graph G_2. It is not hard to check that $G_1 \times G_2$ is a union of two disjoint bipartite graphs, say H_1 and H_2, where the parts of H_1 are $X_1 \times X_2$ and $Y_1 \times Y_2$, and the parts of H_2 are $X_1 \times Y_2$ and $Y_1 \times X_2$.
Suppose that $e = ab$ is an edge of G_1. One can see that the induced subgraph on $\{a\} \times X_2 \cup \{b\} \times Y_2$ in H_1 is isomorphic to G_2. Similarly, if $e' = a'b'$ is an edge of G_2, then the induced subgraph on $X_1 \times \{a'\} \cup Y_1 \times \{b'\}$ in H_1 is isomorphic to G_1.

Therefore we obtain two edge partitions of H_1; H_1 can be partitioned to m_1 copies of G_2 and H_1 can be partitioned to m_2 copies of G_1. Thus by Theorem 2,

$$SO(H_1) \geq m_1 SO(G_2) \text{ and } SO(H_1) \geq m_2 SO(G_1). \tag{4.3}$$

Similarly, we can partition the edges of H_2 as m_1 edge-disjoint copies of G_2 or m_2 edge-disjoint copies of G_1. Therefore by Theorem 2,

$$SO(H_2) \geq m_1 SO(G_2) \text{ and } SO(H_2) \geq m_2 SO(G_1). \tag{4.4}$$

Since $SO(G) = SO(H_1) + SO(H_2)$, by (4.3) and (4.4) the result follows. □

Remark 4. We note that in Theorem 9 the equality holds. For example, let G_1 be a bipartite graph with size m_1 and $G_2 = m_2 K_2$, where m_2 is a positive integer. One can see that $G_1 \times G_2 = 2m_2 G_1$. Thus $SO(G_1 \times G_2) = 2m_2 SO(G_1)$. On the other hand, by Theorem 2, for every graph G, $SO(G) \geq mSO(K_2)$, where m is the size of G. This shows that

$$\max\{2m_2 SO(G_1), 2m_1 SO(G_2), m_2 SO(G_1) + m_1 SO(G_2)\} = 2m_2 SO(G_1) = SO(G_1 \times G_2).$$

Therefore in Theorem 9 the equality holds for the graphs G_1 and G_2.

Conjecture 10. We think that Theorem 9 holds for every two graphs G_1 and G_2.

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