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Research Article

Topological properties of OTIS bijective connection graphs

Paul Immanuel[†], **Berin Greeni**[*]

School of Advanced Sciences, Vellore Institute of Technology, Chennai, 600 127, India
[†]paulimmanuelcd@gmail.com
[*]beringreeni@gmail.com

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Abstract: In the ever-evolving landscape of parallel computing architectures, the demand for innovative interconnection networks is paramount. This paper introduces Optical Transpose Interconnection System (OTIS) - Bijective connection graphs, a subclass of interconnection network designed to address the challenges on scalability, efficiency, and fault tolerance. By merging the strengths of networks, namely, OTIS networks and Bijective connection graphs (BC graphs in brief), we aim to overcome the limitations inherent in individual architectures. This paper presents a comprehensive analysis of Optical Transpose Interconnection System - Bijective connection graphs. We demonstrate superiority over traditional interconnection networks, showcasing their potential to emerge as an interesting candidate for parallel computing. Precisely, in this work, we compute few basic graph theoretical parameters, explored the embedding properties, solved the edge isoperimetric problem, and many associated properties of the proposed class of network.

Keywords: optical transpose interconnection system, bijective connection graph, Hamiltonicity, graph theoretical properties, edge isoperimetric problem.

AMS Subject classification: 05C62, 05C90, 68R10, 94C15, 97P20

1. Introduction

The field of parallel computing relies on efficient interconnection networks to facilitate seamless communication among processors, a critical aspect for achieving optimal performance. The most common interconnection networks include hypercubes, meshes, tori, and more, each with its own advantages and limitations. However, existing networks often face challenges in scalability, adaptability, and fault tolerance. Researchers delve into the development of interconnection architectures, emphasizing

* *Corresponding Author*

the need to strike a balance between factors such as bisection width, fault tolerance, latency, and scalability. The pursuit of these optimized interconnection structures is fueled by the intricate nature of contemporary computing tasks, ranging from large-scale scientific simulations to data-intensive applications. The main disadvantages of the hypercube network include increased complexity and high degree of network connectivity, leading to higher hardware costs with logarithmic diameter. For large networks, the number of connections per vertex becomes large, which can be a drawback in implementation. Hypercubes can suffer from high message latency overhead and low channel utilization. Traditional hypercubes often use static routing algorithms, which can limit the number of available paths and potentially lead to deadlocks. So, many variants were proposed to nullify these disadvantages [8, 9, 11, 46] and so on.
Later in [42], Vaidya et al., proposed a class of graphs and named it Hypercube-Like Networks which includes almost all hypercube variants. In a more general perspective, Fan et al., improvised this class of graphs and proposed a class of graphs known as Bijective connection graphs (BC graphs in brief) denoted by X_n [13]. Bijective connection graphs have the same vertex degree, the highest connectivity (fault tolerance), and diagnosability as hypercubes. Thus, researching on this family of graphs merges the study of some properties of the hypercube and a great many interconnection networks similar to it in structure. Few among the works on X_n include edge-pancyclicity and path-embeddability [12], reliability analysis concerning extra edge-connectivity [50], a-average degree edge-connectivity [47], the relationship between g-extra connectivity and g-extra diagnosability under the MM* model [48], and reliability analysis for components for Bijective connection graphs [16]. These studies provide valuable insight into the structural and reliability aspects of BC graphs.
On the other hand, in the late $90's$ and early 21^{st} century, many interconnection networks were proposed whose building blocks were made up of hypercubes. For example, half hypercube [24], exchanged hypercube [28], dual cube [27], and many more. Then many authors worked on replacing the hypercube with hypercube variants [5, 26] to evaluate sustainability using different connectivity parameters and their spectral properties. More generalized variants of the exchanged hypercube were proposed in [7] and [44]. The former was to nullify the disadvantage of limiting the cross edges between the hypercube clusters, and the latter was to enhance the possibilities of interconnection networks.
The Optical Transpose Interconnection System put forth in [35, 49], introduces a versatile category of scalable interconnection networks known for their high performance. The benefits of optical and electronic technology are seamlessly integrated in this innovative optoelectronic computer architecture [49]. One of the main benefits of using OTIS as an optoelectronic architecture is that it can handle the trade-off efficiently that is associated with free space optical communication [25]. This hybrid interconnection network harnesses the strengths of both optical and electrical communication links between processors. In this design, electrical links are employed for processors in close proximity, optimizing efficiency for short-distance communication. However, processors situated farther apart utilize optical links to leverage the higher speed associated with this technology. This hybrid architecture is commonly referred

to as optoelectronic architectures [49]. This scheme of architecture gives rise to various OTIS networks, including OTIS-Hypercube [38], OTIS-Mesh [43], OTIS-Mesh-of Trees [22], OTIS-Hyper Hexa-Cell [33], and optical chained-cubic tree [31].

Few of the early works regarding the OTIS networks are done in [25, 37, 38]. Krishnamoorthy et al., in [25], indicates that optimal bandwidth, power consumption, minimized system area, and volume are achieved when the cardinality of processors in a group is equal to the total number of groups. Various results on the embeddability of cycles in OTIS networks were performed independently in [10, 20]. Chen et. al., constructed the maximum number of vertex-disjoint paths between two distinct vertices in swapped/OTIS networks, whose objective is to find a solution that optimizes the utilization of network resources while ensuring that the paths between the specified vertices do not share common intermediary vertices [6]. In [34], evaluation of performance on broadcasting is researched for OTIS-Hypercube and OTIS-Mesh. Parallel heuristic local search algorithm was given for two OTIS networks: OTIS Hyper Hexa-cell and OTIS Mesh in [1]. The assessment of topological properties of optoelectronic architectures was computed in [30]. Also, parallel quicksort algorithm for OTIS hyper hexa-cell architecture is given in [2]. Operations such as sorting, routing, data accumulation, prefix sum, consecutive sum, and matrix multiplications have been successfully implemented on OTIS-Mesh [34]. Additionally, load balancing operations were carried out in OTIS-Hypercube [32] and fault-tolerant resolvability of Swapped Optical Transpose Interconnection System is done in [21]. Recently, many novel OTIS architectures such as bi-swapped torus network [17], BSN MOT [18] and petersen-star network [39] have been proposed. Given the extensive research on the OTIS networks and Bijective connection graphs, in this work we aim to combine these concepts by constructing OTIS Bijective connection graphs with the help of both electronic and optic edges where the Bijective connection graphs are made up of electronic edges while these independent graphs are connected by optic edges.

In this paper, we introduce a novel family of graphs called the Optical Transpose Interconnection System - Bijective connection graphs $OTIS - X_n$, which makes use of the advantages of OTIS and Bijective connection graphs X_n. In exploring the properties of Bijective connection graphs, we introduce a class of hybrid networks whose building blocks are Bijective connection graphs. Hypercubes, Mobius cubes, crossed cubes, twisted cubes, locally twisted cubes, spined cubes and Z-cubes are few of the sub classes of Bijective connection graphs. Though there have been few studies on the topological and embeddability properties of different OTIS networks including OTIS-Hypercube [38], we have considered the Bijective connection graphs in which the hypercube itself is one of the basis graphs. In this work, by doing so, we obtain various results for a family of graphs and not an individual graph. Few features and advantages of the considered family of graphs over the other existing OTIS networks are portrayed later in this work later on in tabular form.

The paper is divided into five sections: The next section presents the notation and definitions related to BC graphs, OTIS network, and other graph theoretical parameters. In the third section, we study the basic graph theoretical properties of the proposed network. In the fourth section, we concentrate on embedding path and cycles into

OTIS Bijective connection graphs, and also study an edge isoperimetric problem of OTIS Bijective connection graphs. In the fifth section, we solve many problems as applications of the problem solved in the previous section.

2. Preliminaries

In this section, the necessary notation, definitions related to BC graphs, OTIS network, and other graph theoretical parameters are presented.

2.1. Notation

Let $G = (V_G, E_G)$ be an undirected simple graph. The following notations will be used throughout the paper. A path from a vertex g to a vertex h is denoted by $P_l : g \to i \to \ldots \to h$, where any two vertices in the above sequence are distinct. The vertices g and h are called the end vertices. The length of the path P_l, denoted by $|P_l| = l-1$ is the cardinality of edges in the sequence. If the end vertices are the same, then P_l is called a cycle C_l. The length of the cycle is the cardinality of the edges in the sequence. A graph G has a Hamiltonian path (resp. cycle), if $|P_l| = |V_G|-1$ (resp. $|V_G|$). A graph G is Hamiltonian if there exists a spanning cycle with length $|V_G|$. A graph G is said to be edge pancyclic if there exist cycles of length $3 \leq l \leq |V_G|$. So, if a graph G is edge pancyclic, then G must be Hamiltonian.
Let $|V_G|$ and $|E_G|$ denote the order and size of the graph G, respectively, and $dist_G(g, h)$ denote the length of the shortest path between the distinct vertices g and h in G. The diameter of a graph G denotes the maximum of $dist_G(g, h)$ of all pairs of vertices, denoted by $D(G)$. If the shortest path between two vertices is equal to the diameter, then those vertices are said to be antipodal to each other. Two vertices g and h are said to be adjacent if $(g, h) \in E_G$. The neighborhood set of a vertex g is the set of all vertices adjacent to g in G, represented by $N_G(g) = \{g \in V_G \mid (g, h) \in E_G\}$. The degree of a vertex g in G is denoted by $\deg_G(g)$ which is equal to $|N_G(g)|$. Let $\Delta(G) = \max\{\deg_G(g) \mid g \in V_G\}$ denote the maximum degree of a vertex and $\delta(G) = \min\{\deg_G(g) \mid g \in V_G\}$ denote the minimum degree of a vertex in the graph G.
If $S \subseteq V_G$, let $G[S]$ be the subgraph of G induced by the vertex subset S in G. The neighborhood set of S is defined as $N_G(S) = (\bigcup_{g\in S} N_G(g)) - S$. A vertex cut is a subset of vertices of a connected graph, if deleted disconnects the graph G. The connectivity $\kappa(G)$ of G is defined as the minimum cardinality in all vertex cuts of G. If there exists a subset F, such that $G - F$ is disconnected where the remaining components are not trivial, then F is called a *supercut*. The super connectivity denoted by $\kappa'(G)$, is the minimum cardinality over all such *supercuts* of G. If $S = \{0, 1\}$ in Figure 1, then $N_G(S) = \{2, 3, 4, 5\}$. One of the vertex cuts and supercuts of the given graph is $\{2, 3, 4\}$ and $\{1, 3, 4, 7\}$, respectively.

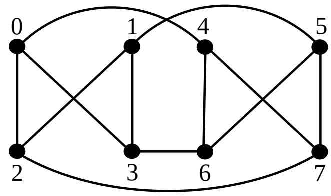


Figure 1. A BC graph

2.2. The class of OTIS BC-graph

Definition 1. [12] Let G be a graph. If $V_G = V_{G_1} \cup V_{G_2}$, $V_{G_1} \neq \emptyset$, $V_{G_2} \neq \emptyset$, and $V_{G_1} \cup V_{G_2} = \emptyset$. We say that there exists a bijective connection between the subsets V_{G_1} and V_{G_2} in G, denoted by $V_{G_1} \stackrel{G}{\leftrightarrow} V_{G_2}$, if G satisfies the two conditions:

1. For every $h_1 \in V_{G_1}$, there exists an unique $h_2 \in V_{G_2}$ such that $(h_1, h_2) \in E(G)$; and
2. For every $h_1 \in V_{G_2}$, there exists an unique $h_2 \in V_{G_1}$ such that $(h_1, h_2) \in E(G)$.

An $n-$dimensional Bijective connection graph (BC graph), denoted by X_n, is an $n-$ regular graph with 2^n vertices and $n(2^{n-1})$ edges. The set of all $n-$dimensional BC graphs is called the family of $n-$dimensional BC graphs, denoted by $\mathcal{L}_n$. The recursive definitions of X_n and $\mathcal{L}_n$ are as follows:

Definition 2. [12] The $1-$dimensional BC graph X_1 is a complete graph on two vertices. The family of the $1-$dimensional BC graph is defined as $\mathcal{L}_1 = \{X_1\}$. Let G be a graph. G is an $n-$dimensional BC graph, denoted by X_n if there exists $V_{G_0}, V_{G_1} \subset V_G$ such that the following two conditions hold:

1. $V_G = V_{G_0} \cup V_{G_1}, V_{G_0} \neq \emptyset, V_{G_1} \neq \emptyset$, and $V_{G_0} \cap V_{G_1} = \emptyset$; and
2. $V_{G_0} \stackrel{G}{\leftrightarrow} V_{G_1}$, $G[V_{G_0}] \in \mathcal{L}_{n-1}$, and $G[V_{G_1}] \in \mathcal{L}_{n-1}$.

The family of the $n-$dimensional BC graphs is defined as $\mathcal{L}_n = \{G \mid G$ is an $n-$dimensional BC graph$\}$. A pictorial representation of recursive nature of a BC graph of dimensions $1, 2$ and 3 is given in Figure 2. Figure 3 presents few networks from the family of Bijective connection graphs $\mathcal{L}_3$.

Definition 3. Let $OTIS - G$ denote the OTIS network. Then $OTIS - G = (V_G, E_G)$ network is an undirected graph given by:

1. $V_{OTIS-G} = \{\langle g, h\rangle \mid g, h \in V_G\}$ and
2. $E_{OTIS-G} = \{(\langle g, h_1\rangle, \langle g, h_2\rangle) \mid g \in V_G \textit{ and } (h_1, h_2) \in E_G\} \cup \{(\langle g, h\rangle \sim \langle h, g\rangle) \mid g, h \in V_G \textit{ and } g \neq h\}$.

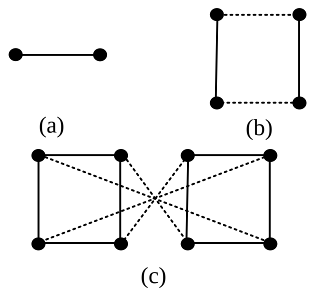


Figure 2. Recursive construction of a BC graph: (a) Complete graph on 2 vertices which is a X_1 (b) A cycle on 4 vertices which is a X_2 (c) X_3 with two X_2s where the dashed lines represent the edges between two X_{n-1}s

Here, G is the basis graph of $OTIS-G$. If G has n vertices, then $OTIS-G$ is generated by n vertex disjoint sub networks $G_1, G_2, \ldots, G_n$, called clusters. Each of these clusters is isomorphic to the basis graph G. A vertex $\langle g, h\rangle$ in $OTIS-G$ corresponds to the vertex with address h in the cluster g. So, g refers to the group address while h refers to the processor address in a vertex $\langle g, h\rangle$. An intragroup edge of the form $(\langle g, h_1\rangle, \langle g, h_2\rangle)$ corresponds to an electronic edge, which means that an intragroup edge is an edge that is present in the considered basis graph.
An intergroup edge of the form $(\langle g, h\rangle \sim \langle h, g\rangle)$ corresponds to an optic edge, which means that an optic edge acts as the bridge between any two basis graph copies. This definition characterizes a wide class of networks, and for any known basis graph G, a corresponding $OTIS-G$ can be constructed.
In Figures 4, 5, the basis graphs are hypercube and crossed cube of dimension 3, respectively. Similarly, for any Bijective connection graph, OTIS network can be constructed. Hence, the proposed novel network is the class of OTIS representation of every BC graph. Let $OTIS-X_n$ denote the OTIS network whose basis graph can be any BC graph.

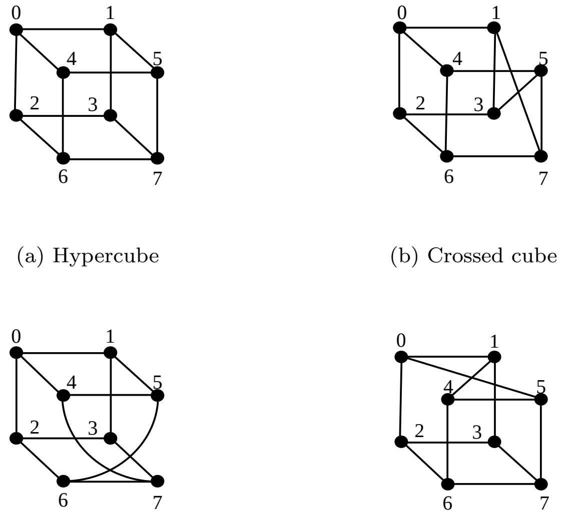
3. Basic graph theoretical properties of OTIS BC-graph

In this section, we examine several graph theoretical properties of the proposed class of OTIS networks.

Remark 1. The number of vertices in $OTIS-X_n$ is given by, $|V_{OTIS-X_n}| = 2^{2n}$.

Proposition 1. *The number of edges in $OTIS-X_n$ is given by,* $|E_{OTIS-X_n}| = 2^n(n2^{n-1}) + 2^{2n-1} - 2^{n-1}$.

Proof. By definition of $OTIS-X_n$, the edges can be separated into two types E_1



(a) Hypercube (b) Crossed cube

(c) 0-Mobius cube (d) 1-Mobius cube

Figure 3. **Few examples of Bijective connection graphs**

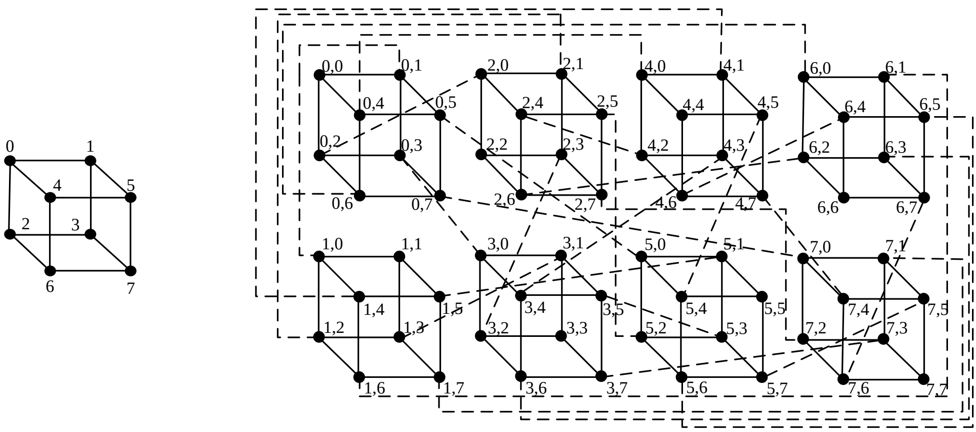


Figure 4. (a) Hypercube of dimension 3 (b) The OTIS network of dimension 3 whose basis graph is hypercube where normal and dash edges represent the electronic (Bijective connection, intragroup) edges and optic (OTIS, intergroup) edges, respectively

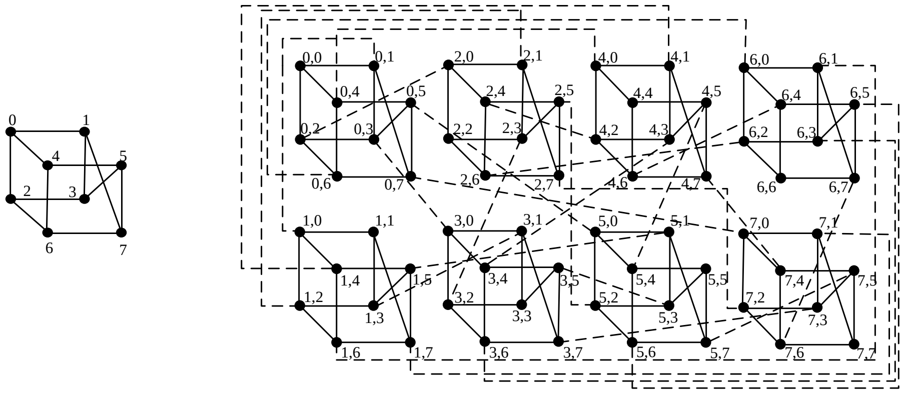


Figure 5. (a) Crossed cube of dimension 3 (b) The OTIS network of dimension 3 whose basis graph is crossed cube where normal and dash edges represent the electronic (Bijective connection, intragroup) edges and optic (OTIS, intergroup) edges, respectively

Table 1. The difference between the cardinality of electronic and optic edges of the $OTIS - X_n$

Dimension	$\|V(OTIS - X_n)\|$	Edges		
n		Electronic	Optical	Total
3	64	96	28	124
4	256	512	120	632
5	1024	2560	496	3056
6	4096	12288	2016	14304

and E_2: The edge sets E_1 and E_2 correspond to the edges in the basis graph G and the OTIS edges, respectively. In otherwords, the edges in E_1 are the electronic edges and the edges in E_2 are the optic edges. There exists $|V_{X_n}|$ disjoint X_ns. So, we have $|E_1|$ = number of clusters $\times$ number of edges in the basis BC graph of dimension n $= 2^n(n2^{n-1})$. OTIS edges are the edges that connect a cluster to a different cluster. Note that $E_2 = \{(\langle g,h\rangle, \langle h,g\rangle) |$ only when $g \neq h\}$. Therefore, no optic edge is incident on the vertex of the form $\langle g,h\rangle$ where $g = h$ and the cardinality of these vertices is 2^n. Then, the cardinality of such non existing edges is 2^{n-1}. So, $|E_2| = 2^{2n-1} - 2^{n-1}$. Hence, $|E_{OTIS-X_n}| = |E_1| + |E_2| = 2^n(n2^{n-1}) + 2^{2n-1} - 2^{n-1}$. See Table 1 for the cardinality of both electronic and optic edges in $OTIS - X_n$. □

Proposition 2. *The degree of a vertex $\langle g,h\rangle$ in $OTIS - X_n$ is given by,*

$$deg_{OTIS-X_n}\langle g,h\rangle = \begin{cases} n & when\ g = h \\ n+1 & when\ g \neq h \end{cases}.$$

Proof. The $OTIS - X_n$ is composed of disjoint clusters of X_n connected by the OTIS edges. According to the definition of $OTIS - X_n$, the vertex $\langle g,h\rangle \in V_{X_n}$

does not have an OTIS edge incident on it when $g = h$. Hence, the number of edges incident on the vertex $\langle g, h\rangle$ is the number of edges incident on a vertex in a X_n which is n and the number of edges incident on the vertex $\langle g, h\rangle$ with $g \neq h$ is $n + 1$, since there exists exactly an OTIS edge of the form $(\langle g, h\rangle \sim \langle h, g\rangle)$. □

Lemma 1. *The connectivity, edge connectivity and minimum degree of $OTIS - X_n$ is given by, $\kappa(OTIS - X_n) = \lambda(OTIS - X_n) = \delta(OTIS - X_n) = n$.*

Proof. For a vertex $\langle g, h\rangle$ with $g = h$ to be disconnected from the rest of the graph, the cardinality of vertices that should be deleted equals the cardinality of edges incident on the vertex, i.e., $|N_{OTIS-X_n}(\langle g, h\rangle)|$. As $deg_{OTIS-X_n}\langle g, h\rangle = n$ for the considered vertex, $|N_{OTIS-X_n}(\langle g, h\rangle)| = n$ is the vertex connectivity of $OTIS - X_n$. Clearly, $\kappa(OTIS - X_n) \leq \lambda(OTIS - X_n)$. If F is a set with the least number of edges whose deletion leaves the graph disconnected, then $\delta(OTIS - X_n) = |F|$, but according to the definition of edge connectivity, $\lambda(OTIS - X_n) = |F|$. From Menger's theorem [45], there exist at least k parallel paths between any two distinct vertices in a graph with connectivity k. Hence, there should exist at least $\delta(OTIS - X_n)$ disjoint paths. The theorem suffices if $\lambda(OTIS - X_n) \leq \kappa(OTIS - X_n)$ is proved. From Menger's theorem, $\kappa(OTIS - X_n) \geq \delta(OTIS - X_n)$. As $\delta(OTIS - X_n) = |F| = \lambda(OTIS - X_n)$, we have, $\kappa(OTIS - X_n) \geq \delta(OTIS - X_n) = \lambda(OTIS - X_n)$, which implies $\kappa(OTIS - X_n) \geq \lambda(OTIS - X_n)$. Hence, the statement holds. □

Lemma 2. *The diameter of $OTIS - X_n$ is given by, $D(OTIS - X_n) = 2D(X_n) + 1$.*

Proof. The proof suffices if we prove that there exists a path of length at least $2D+1$ and a path of length at most $2D + 1$ between the antipodal vertices. Let $\langle g_1, h_1\rangle$ and $\langle g_2, h_2\rangle$ be antipodal vertices. If they are antipodal then $g_1 \neq g_2$, i.e., there should exist an OTIS edge in the path and that h_1 and h_2 should lie in different clusters. The path then has to traverse from some h_1 to h_3 in the cluster g_1 of length equal to diameter of the considered basis graph which is a BC graph. Let $(\langle g_1, h_1\rangle, \langle g_2, h_4\rangle)$ be the OTIS edge and from $\langle g_2, h_4\rangle$ the path has to traverse from h_4 to h_2 in the cluster g_2 of length equal to diameter of the considered basis graph which is a BC graph. So, the length of the path between the antipodal vertices $\langle g_1, h_1\rangle$ and $\langle g_2, h_2\rangle$ must be at least $2D + 1$.
Now, assume $2D + 2$ to be the length of the path between two antipodal vertices. If the length of the path between two antipodal vertices must be $2D + 2$ then the two edges must be OTIS edges which is not the case because any cluster in an OTIS network is connected to any other cluster. Thus, there is no need for 2 or more OTIS edges. Therefore, the shortest path between two antipodal vertices is at most $2D+1$. Hence, the proof.

□

An illustration of the shortest path between two antipodal vertices in $OTIS - X_2$ is illustrated in Figure 6.

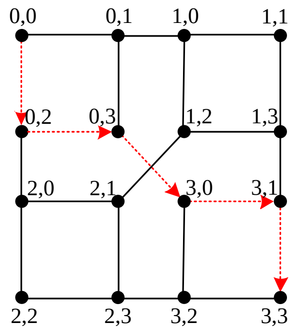


Figure 6. **Illustration of calculation of diameter on** $OTIS - X_2$

Remark 2. $OTIS - X_n$ is not Eulerian.

As $OTIS - X_n$ is biregular, any vertex $\langle g_i, h_i\rangle$ is of degree n or $n + 1$.

4. Topological properties of OTIS BC graphs

In this section we explore the graph embeddability and examine the possibilities and constraints of embedding literaturely rich graphs. The purpose of this investigation is to identify the underlying structures and characteristics that determine whether the paths and cycles can be embedded into the OTIS BC graphs.

4.1. Embeddability of OTIS BC graphs

The OTIS BC graphs contain both odd and even cycles without depending on the basis graph with no odd or even cycles.

Remark 3. There exists only one OTIS edge $(\langle g_1, h_1\rangle \sim \langle g_2, h_2\rangle)$ where $g_1 \neq g_2$ such that no other OTIS edge of form $(\langle g_1, h_3\rangle \sim \langle g_2, h_4\rangle)$ where $h_1 \neq h_3$ and $h_2 \neq h_4$ is possible. Moreover, the OTIS edges act as a bridge between independent clusters of basis graphs. Hence, the maximum number of edges induced for any l vertices for $2 \leq l \leq 2^{2n}$ should be along the basis graph in any $OTIS - G$.

Lemma 3. *There exists a path P_l on l vertices for $2 \leq l \leq 3 \cdot 2^n$ in an $OTIS - X_n$.*

Proof. To prove the statement, it will suffice to prove the presence of a path of length $3 \cdot 2^n - 1$. Let $(\langle g_1, h_1\rangle \sim \langle g_2, h_2\rangle)$ denote the OTIS edge between the vertices $\langle g_1, h_1\rangle$ and $\langle g_2, h_2\rangle$. If $\langle g_1, h_1\rangle$ is a vertex in the cluster g_1 with processor address

h_1, then there exists a path on $2^n - 1$ vertices in the cluster g_1, between the vertices, $\langle g_1, h_1\rangle$ and $\langle g_1, h_2\rangle$ (say). So the traversal will be of the form,

$$\langle g_1, h_1\rangle \quad \underbrace{\rightarrow \ldots \rightarrow}_{path\ of\ length\ 2^n-1} \quad \langle g_1, h_2\rangle$$

Let $g_1 \neq h_2$. Then, from $\langle g_1, h_2\rangle$ there obviously exists an OTIS edge to some $\langle g_2, h_3\rangle$. Then traversal including the traversal inside the g_2 cluster will be of form

$$\langle g_1, h_1\rangle \quad \underbrace{\rightarrow \ldots \rightarrow}_{path\ of\ length\ 2^n-1} \quad \langle g_1, h_2\rangle \sim \langle g_2, h_3\rangle \quad \underbrace{\rightarrow \ldots \rightarrow}_{path\ of\ length\ 2^n-1} \quad \langle g_2, h_4\rangle$$

From Remark 3, there can exist only one OTIS edge between any two clusters. So, the edge $\langle g_1, h_2\rangle \sim \langle g_2, h_3\rangle$ is unique and there cannot be another edge from cluster g_1 to cluster g_2. Let $g_2 \neq h_4$. Then, there should exist an OTIS edge $\langle g_2, h_4\rangle \sim \langle g_3, h_5\rangle$. Then the traversal including the traversal inside the cluster g_3 will have the form,

$$\langle g_1, h_1\rangle \quad \underbrace{\rightarrow \ldots \rightarrow}_{path\ of\ length\ 2^n-1} \quad \langle g_1, h_2\rangle \sim \langle g_2, h_3\rangle \quad \underbrace{\rightarrow \ldots \rightarrow}_{path\ of\ length\ 2^n-1} \quad \langle g_2, h_4\rangle \sim \langle g_3, h_5\rangle \quad \underbrace{\rightarrow \ldots \rightarrow}_{path\ of\ length\ 2^n-1} \quad \langle g_3, h_6\rangle.$$

Hence, there exists a path of length $3 \cdot 2^n - 1$ in $OTIS - X_n$. □

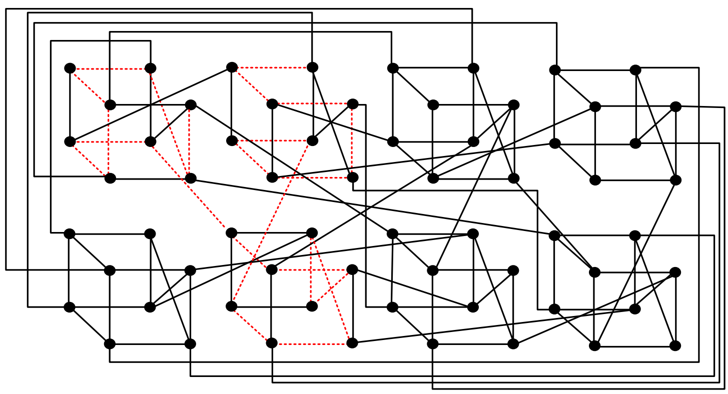


Figure 7. **Presence of the path of length** $3 \cdot 2^n - 1$

See Figure 7 for an illustration of the presence of such a path of above said length in $OTIS - X_3$. Although the Lemma 3 seems trivial, it might not be so when the length of path, $|P_l| > 3 \cdot 2^n - 1$, as there might exist an OTIS edge, $\langle g_1, h_1\rangle \sim \langle g_3, h_6\rangle$. Theorems 1 and 3 are useful in proving the presence of a Hamiltonian cycle in $OTIS - X_n$ in the upcoming results. Now, we move on to prove the presence of a cycle of length $(2^n)^2$. Then automatically, the above result can be extended in the sense that a path P_l on l vertices for $1 \leq l \leq (2^n)^2 - 1$ is present in $OTIS - X_n$.

Theorem 1. [12] *For every integer $n \geq 4$, and any $X_n \in \mathcal{L}_n$ with $V_{G_0} \overset{G}{\leftrightarrow} V_{G_1}$, suppose that the following three conditions are satisfied:*

1. *For every $\{h_1, h_2\} \in E_{X_n}$ with $h_1 \in V_{G_0}$ and $h_2 \in V_{G_1}$, there is a cycle C_l of length l such that $\{h_1, h_2\}$ is in C_l in X_n for every integer $l \in \{4, 5\}$;*
2. *For every integer $i \in \{0, 1\}$ and $h_1, h_2 \in V_{G_i}$ with $h_1 \neq h_2$, $dist(X_n[V_{G_i}], h_1, h_2) = dist(X_n, h_1, h_2)$;*
3. *For $h_1, h_2 \in V_{Gi}$ with $h_1 \neq h_2$ and two integers l' and i with $dist(X_n[V_{G_i}], h_1, h_2) + 2 \leq l' \leq 2^{n-1} - 1$ and $i \in \{0, 1\}$, there exists a path of length l' between h_1 and $h_2 \in X_n[V_{G_i}]$.*

Then, there exists a path of length l between h_1 and h_2 in X_n for $h_1, h_2 \in V_{X_n}$ with $h_1 \neq h_2$ and every integer l with $dist(X_n, h_1, h_2) + 2 \leq 2^n - 1$.

Theorem 2. [12] *For every integer $n \geq 4$, and every $X_n \in \mathcal{L}_n$ with $V_{G_0} \overset{G}{\leftrightarrow} V_{G_1}$, if the three conditions in Theorem 1 hold, then X_n is edge pancyclic.*

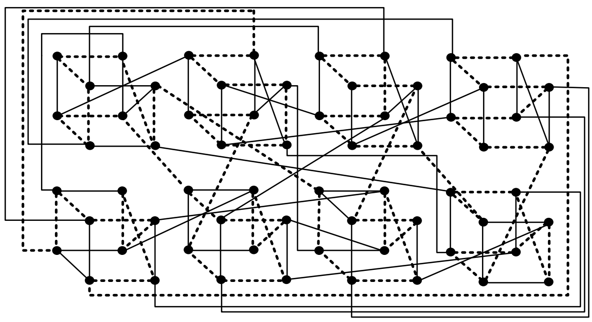


Figure 8. Hamiltonicity of $OTIS - X_n$ is illustrated using dashed lines which forms a cycle of length 64 in $OTIS - X_3$

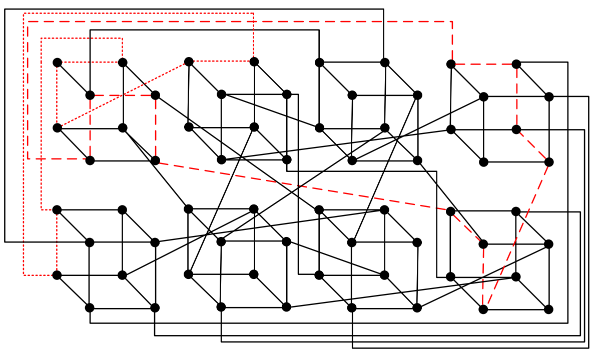


Figure 9. An illustration of presence of odd cycles in $OTIS - X_n$ even when the basis graph does not contain any odd cycle

Theorem 3. [10, 36] *If G is Hamiltonian, then $OTIS - G$ is Hamiltonian.*

Theorem 4. *The $OTIS - X_n$ graph is Hamiltonian if X_n satisfies all three conditions of Theorem 1.*

Proof. The proof is straightforward by Theorems 1 and 3. An illustration of Hamiltonicity of $OTIS - X_n$ is shown in Figure 8. □

Lemma 4. *The $OTIS - X_n$ consists of odd cycles irrespective of the presence of odd cycles in the basis graph.*

Proof. According to the definition of BC graphs, X_2 is a cycle on four vertices. As BC graphs are recursive in nature, X_n should contain 2^{n-1} disjoint cycles of length four. Let $\langle g_1, h_1\rangle$ and $\langle g_1, h_2\rangle$ be two vertices in a cycle C_4 at distance two. Then, there should exist two vertices of the form $\langle h_1, g_1\rangle$ and $\langle h_2, g_1\rangle$ in the clusters h_1 and h_2, respectively. As the cycle should traverse at least three OTIS edges to form a cycle with OTIS edges by Remark 3, there should exist exactly three clusters. It is easy to find an edge independently in both clusters h_1 and h_2 of form $(\langle h_1, g_1\rangle, \langle h_1, h_2\rangle)$ and $(\langle h_2, g_1\rangle, \langle h_2, h_1\rangle)$. Notice that $\langle h_1, h_2\rangle$ and $\langle h_2, h_1\rangle$ should be connected by an OTIS edge. Hence, $OTIS - X_n$ has cycles of odd lengths, even when the basis graph X_n does not contain an odd cycle. □

For illustration, we have considered the hypercube as the basis graph, which does not contain odd cycles of any length; however, there exists a cycle of odd length in $OTIS - X_n$, with the help of OTIS edges (at least three edges), in Figure 9. The normal mini-dashed edges and dark dashed edges in the Figure 9 showcase the presence of odd cycles of length 7 and 11, respectively.

Remark 4. $OTIS - X_n$ is not bipartite.

From Lemma 4, cycles of odd length exists, even when the basis graph does not contain cycles of odd lengths in $OTIS - X_n$. So, $OTIS - X_n$ cannot be bipartite.

4.2. Induced subgraph problem on OTIS BC graphs

Theorem 5. [41] *For any $n-$dimensional BC graph X_n, the maximum number of edges joining vertices from a set of m vertices is given by $|E(X_n[m])| = \sum_{i=0}^{r-1}(\frac{l_i}{2} + i)2^{l_i}$ where $m = \sum_{i=0}^{r-1} 2^{l_i}$ for some non negative integers r and $l_0 > l_1 > \ldots > l_{r-1}$ where $n \geq 1$ and $1 \leq m \leq 2^n$.*

Theorem 6. *The maximum number of edges induced by m vertices of an $OTIS - X_n$ where $m = a \cdot 2^n + b$ for $0 \leq a \leq 2^n$, $0 \leq b \leq 2^n - 1$ is given by,*

$$|E(OTIS{-}X_n[m])| = \begin{cases} a \times |E(X_n[2^n])| + |E(X_n[b])| + a(a-1)/2 & \textit{when } a = 0 \textit{ or } b = 0 \\ a \times |E(X_n[2^n])| + |E(X_n[b])| + x + a(a-1)/2 & \textit{when } a, b \geq 1 \end{cases}$$

where $x = min\{a, b\}$.

Proof. The proof is divided into two cases.

Case 1. When a or $b = 0$.

Case 1.1. When $a = 0$, there does not exist a complete X_n and hence $|E(OTIS - X_n[m])| = |E(X_n[b])|$.

Case 1.2. When $b = 0$, there exist exactly a complete copies of X_n in m. In addition, the term $a(a-1)/2$, denotes the number of OTIS edges between complete copies of X_n. We prove this by induction on a. Assume it to be true for $1 \leq a \leq 2^n - 1$. When $a = 2^n$, we have $\frac{2^n(2^n-1)}{2}$ edges between disjoint copies of X_n, which is exactly the number of OTIS edges present in an $OTIS - X_n$, as $|E_2| = 2^{2n-1} - 2^{n-1}$. Hence, $|E(OTIS - X_n)[m])| = a|E(X_n[2^n])| + a(a-1)/2$.

Case 2. When $1 \leq a, b \leq 2^n - 1$.

From the above case, the expression $a|E(X_n[2^n])| + |E(X_n[b])| + a(a-1)/2$ is justified. Consider i^{th} cluster, a BC graph, it can have x OTIS edges if and only if the traversing on m vertices is done on exactly x disjoint clusters before the i^{th} cluster. As a and b represent the complete BC graphs in m and the vertices present in the incomplete BC graph, respectively, x should be the minimum between a and b. Thus, $|E(OTIS - X_n[m])| = a|E(X_n[2^n])| + |E(X_n[b])| + x + a(a-1)/2$ where $x = min\{a, b\}$. □

See Figure 10 which depicts the pattern of ordering the vertices which yields induced subgraph of $OTIS - X_n$. In addition, the specific ordering that solves the induced subgraph problem is obtained by a simple function, $f\langle g, h\rangle = 2^n g + h$.

5. Applications

In this section, we compute the minimum linear arrangement, bisection width, and super (edge) connectivity of $OTIS - X_n$, as an application of finding the induced subgraph of $OTIS - X_n$.

5.1. Minimum linear arrangement of OTIS BC graphs

Harper's work in [19] initially introduced the assignment of vertices with specific numerical patterns to minimize the average absolute error in transmission, leading to the formulation of what is now known as the minimum linear arrangement problem ($MinLA$) and is $NP-$Complete [14]. A minimum linear arrangement of a graph refers to a linear ordering of its vertices in such a way that the sum of the lengths of the edges is minimized. In other words, it is a way of arranging the vertices along a line such that the total distance between adjacent vertices (corresponding to edge lengths) is minimized. In tasks that involve sequential processing of vertices, the process of linear ordering may simplify the implementation. This is why we use specific ordering

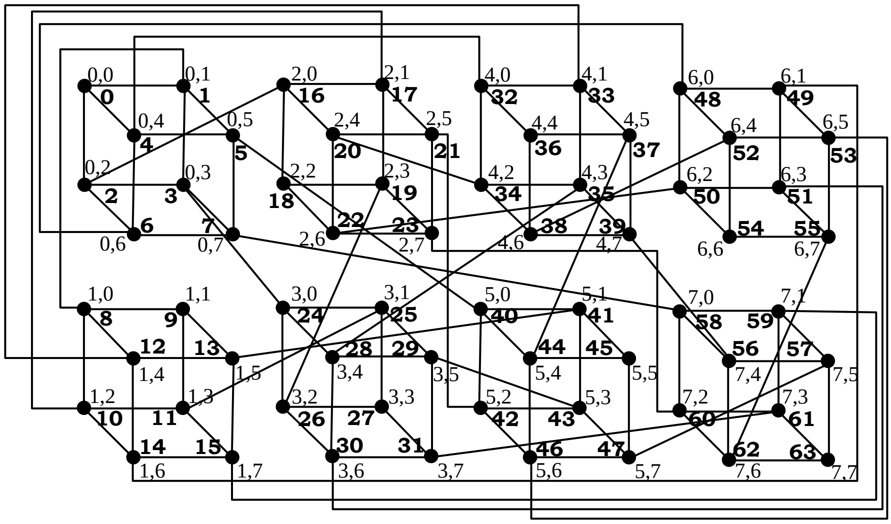


Figure 10. $OTIS - X_3$ with the ordering in bold italics represents the ordering of vertices which yield induced subgraph while the normal font represents the generic ordering of vertices

on the vertices of the graph which yields the maximum subgraph and then proceed with that particular ordering to achieve the optimal linear arrangement. Readers are referred to [15, 40] for works related to minimum linear arrangement.

Theorem 7. [23] *The minimum linear arrangement of BC graph X_n is given by,* $MinLA(X_n) = 2^{n-1}(2^n - 1)$.

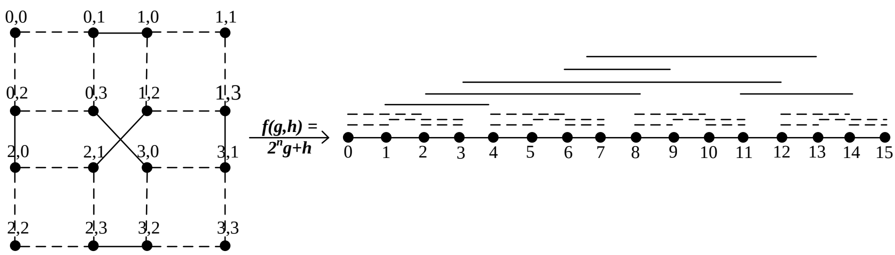


Figure 11. Minimum linear arrangement of $OTIS - X_2$ where dashed lines and bold lines represent the linear arrangement yielded by intra edges inside X_2 and inter edges connecting disjoint copies of X_2, respectively

Theorem 8. *The minimum linear arrangement of $OTIS - X_n$ is given by,*

$MinLA(OTIS-X_n) = (2^{2n-1}-2^{n-1})(2^n) + (2^n-1)\big(\frac{(2^n-1)(2^n)}{2} + \frac{(2^n-2)(2^n-1)}{2} + \ldots + 1\big).$

Proof. The minimum linear arrangement of X_n graphs was studied and proved to be $2^{2n-1}-2^{n-1}$. As the clusters of $OTIS-X_n$ are all X_n graphs and there exists 2^n disjoint clusters, the minimum linear arrangement of these clusters add up to $(2^{2n-1}-2^{n-1})(2^n)$. As mentioned earlier, we use the ordering which yields the maximum subgraph and thus the linear arrangement imposed by the OTIS edges should also be minimum. Then, every OTIS edge from the 1^{st} cluster to all other clusters will yield, $(2^n-1)1+(2^n-1)2+\ldots+(2^n-1)2^n-1$. Recursively concatenating the linear arrangement imposed by every cluster until $(2^n-1)^{th}$ cluster, $\big(((2^n-1)1+(2^n-1)2+\ldots+(2^n-1)(2^n-1))+((2^n-1)1+(2^n-1)2+\ldots+(2^n-1)(2^n-2))+\ldots+(2^n-1)\big)$ is the minimum linear arrangement imposed by all the OTIS edges. Therefore, the minimum linear arrangement of $OTIS-X_n$ is minimum and is given by, $(2^{2n-1}-2^{n-1})(2^n)+(2^n-1)\big(\frac{(2^n-1)(2^n)}{2}+\frac{(2^n-2)(2^n-1)}{2}+\ldots+1\big)$.

□

A pictorial representation of minimum linear arrangement of $OTIS-X_2$ is presented in Figure 11.

5.2. Bisection width of OTIS BC graphs

There are many results concerning the bisection width of regular graphs. In [4], a straightforward technique for finding bisection width is given for any regular graph G, by $bw(G) = \theta_G(\lfloor \frac{V_G}{2} \rfloor)$, where $\theta_G(m)$ stands for the minimum number of edges leaving the m vertices of a graph G. As $OTIS-X_n$ is biregular, finding the bisection width is not straightforward.

Remark 5. The bisection width of X_n is given by, $bw(X_n) = 2^{n-1}$ for $n \geq 1$.

The statement is true from the relation given above.

Theorem 9. *The bisection width of an $OTIS-X_n$ graph is given by, $bw(OTIS-X_n) = 2^{2n-2}$ for $n \geq 2$.*

Proof. When $n = 2$, it is evident from Figure 11 which has $OTIS-X_2$, that four OTIS edges need to be deleted to obtain two disconnected components with equal size, which is the minimum. When $n \geq 3$. Every cluster in $OTIS-X_n$ is a X_n that is n-regular and has 2^n vertices and $n2^{n-1}$ edges. In order to find the bisection width, the graph $OTIS-X_n$ should become two equal sized components with 2^{2n-1} vertices each. Intuitively, there can be three cases in this regard. Deleting only X_n edges: But doing so will never give two equal sized components as every disjoint X_n is connected to each other by 2^n-1 OTIS edges. Deleting a few X_n edges and OTIS edges: Although there may a possibility of obtaining two equal sized components, the number of edges deleted will be very large as it includes X_n edges (by Remark 5), which leaves us with the last case. Deleting only OTIS edges: As we only concentrate on deleting OTIS edges, the two equal sized components which has 2^{2n-1} vertices each, will contain 2^{n-1} disjoint X_ns connected by $2^{n-1} - 1 + 2^{n-1} - 2 + \ldots + 1$ OTIS edges. By Proposition 1, we know that the cardinality of $OTIS - X_n$ edges is $2^{2n-1} - 2^{n-1}$. So, subtracting $2^{n-1} - 1 + 2^{n-1} - 2 + \ldots + 1$ OTIS edges in both the equal sized components, we arrive at $2^{2n-1} - 2^{n-1} - 2(2^{n-1} - 1 + 2^{n-1} - 2 + \ldots + 1)$ which is equal to 2^{2n-2}.

□

In Figure 12, 16 edges has to deleted for the graph $OTIS - X_3$ to be disconnected into equal halves.

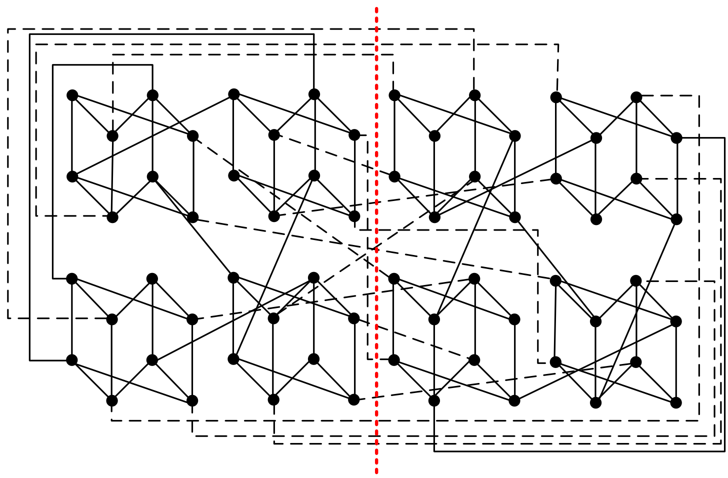


Figure 12. Bisection width of $OTIS - X_3$ where the dashed edges are the minimum number of OTIS edges that needs to be deleted in order for the $OTIS - X_3$ to be cut into two equal halves

5.3. Super connectivity of OTIS BC graphs

An edge should be selected as one of the components in order to obtain the least cardinality of the vertices to be deleted, because the expression in Theorem 6 is strictly increasing as m increases. Also, according to the definition of super connectivity of a graph, there can exist components that are only non trivial.

Lemma 5. *The super connectivity of $OTIS - X_n$ graph is given by, $\kappa'(OTIS - X_n) = 2n - 1$ for $n \geq 2$.*

Proof. If F is the set of vertices whose removal leaves the graph disconnected with no components having size one, then $|F|$ which has the least number of vertices out of all possibilities is the super connectivity of a graph G. Let

Table 2. A comprehensive comparative analysis of the $OTIS - X_n$ with other OTIS networks

Interconnection networks	$\|V_G\|$	Diameter	Bisection width	$\{min, max\}$ degree
$OTIS - X_n$	$(2^n)^2$	$2D + 1$	2^{2n-2}	$\{n, n + 1\}$
$OTIS - S_n$ [3]	$(\|n!\|)^2$	$2\lfloor 1.5(n - 1)\rfloor + 1$	$\frac{(M^2-M)}{2} + N^2$	$\{n - 1, n\}$
$OTIS - M_{n\times n}$ [43]	n^2	$4\sqrt{(n)} - 3$	Refer [29]	$\{2, 5\}$
$OTIS - HHC_n$ [33]	$6 \times (2_h^{d-1})$	$2 \times d_h + 3$	$(\frac{6\times 2_h^{d-1}}{2})^2$	$\{d_h + 2, d_h + 3\}$
$OCCT(h, d, lv)$ [31]	$g \times 2^d$	Refer [31]	$2^d \times (H + 1.5)+$ $((2^{lv}/2)^2 - 1) \times 2^d)$	Refer [31]

$(\langle g_1, h_1\rangle, \langle g_2, h_2\rangle)$ edge be a component after deleting $|F|$ vertices. Now, choosing an edge whose $|N_G(\langle g_1, h_1\rangle, \langle g_2, h_2\rangle)|$ is minimum will give the super connectivity of $OTIS - X_n$ graph. From Proposition 2, there exist only two types of edges: $(\langle g_1, h_1\rangle, \langle g_2, h_2\rangle)$ with $deg_{OTIS-X_n}\langle g_1, h_1\rangle = n$ and $deg_{OTIS-X_n}\langle g_2, h_2\rangle = n+1$, or with $deg_{OTIS-X_n}\langle g_1, h_1\rangle = deg_{OTIS-X_n}\langle g_2, h_2\rangle = n+1$. Obviously choosing an edge inside any cluster with one vertex having degree n and other vertex having degree $n+1$ will have the least number of neighbors. Then, $|N_G(\langle g_1, h_1\rangle, \langle g_2, h_2\rangle)| = n - 1 + n = 2n - 1$. Choosing any other edge to be a component will yield more or exactly the same number of neighbor vertices adjacent to the vertices in the considered edge. Hence, the proof. □

Lemma 6. *The super edge connectivity of $OTIS-X_n$ graph is given by $\lambda'(OTIS-X_n) = 2n - 1$ for $n \geq 2$.*

Proof. From Remark 3 and Definition 3, there exists no 3 cycle, meaning, every $2n - 1$ edges incident on $2n - 1$ vertices are distinct. Hence $\kappa'(OTIS - X_n) = \lambda'(OTIS - X_n) = 2n - 1$. □

To illustrate the superiority of $OTIS - X_n$ over other OTIS networks and compare with other OTIS networks we have provided two tables Table 2 and Table 3. In Table 3 we have demonstrated the diameter in numerical ranges of several OTIS networks by considering their dimensions. $OTIS - X_n$ outperforms the other OTIS networks in terms of diameter, although the cardinality of edge set of $OTIS - X_n$ increases exponentially. $OTIS - X_n$ is the only network that relies on the diameter of the Bijective connection graph used as the basis graph. With correspondence to the bisection width, $OTIS - X_n$ performs reasonably well when the cardinality of the edge set of the other OTIS networks are taken into account.

6. Concluding remarks

In this study, we introduce a novel class of graphs called OTIS BC graphs in which we have utilized the Bijective connection networks as the basis graph for OTIS network.

This variation improves the fault tolerance, latency, and scalability of the considered basis graphs, which was evident through the following studies: First, we examined the fundamental graph theoretical characteristics of $OTIS-X_n$. Next, we investigate the problem of embedding paths and cycles into $OTIS-X_n$ of different lengths. In addition, we solved the induced subgraph problem, which helped us to compute the minimum linear arrangement, bisection width, and the super (edge) connectivity of $OTIS-X_n$. It would be worth studying the fault tolerant measures of the $OTIS-X_n$ which includes a variety of connectivity parameters. In our research, we have considered $OTIS-X_n$ to be our host graph and embedded path and cycle into them. It would be an interesting problem to solve the embedding problem by considering $OTIS-X_n$ as the guest graph.

Table 3. A comprehensive comparative analysis of the diameter of $OTIS-X_n$ with other OTIS networks

$OTIS-X_n$			$OTIS-S_n$			$OTIS-M_{n\times n}$			$OTIS-HHC_n$			$OCCT(h,d,lv)$		
n	$\lvert V_G\rvert$	Diameter	n	$\lvert V_G\rvert$	Diameter	n	$\lvert V_G\rvert$	Diameter	n	$\lvert V_G\rvert$	Diameter	(h,d,lv)	$\lvert V_G\rvert$	Diameter
1	4	3	1	1	1	1	1	1	1	36	5	$(1,2,3)$	8	3
2	16	5	2	4	3	4	16	5	2	144	7	$(1,3,3)$	16	4
3	64	5	3	36	7	9	81	9	3	576	9	$(2,2,3)$	8	5
4	256	7	4	576	9	16	256	13	4	2304	11	$(2,3,3)$	16	6
5	1024	7	5	14400	13	25	625	17	5	9216	13	$(2,4,3)$	32	7

Conflict of Interest: The authors declare that they have no conflict of interest.

Data Availability: Data sharing is not applicable to this article as no data sets were generated or analyzed during the current study.

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