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Research Article

Traversibility of subspace based nonzero component graph of vector spaces over finite fields

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Abstract: Let $\mathbb{V}$ be an n-dimensional vector space over a field $\mathcal{F}$ having basis $\mathcal{B} = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ and W be an m dimensional subspace of $\mathbb{V}$ with basis $\{\alpha_{w_1}, \alpha_{w_2}, \ldots, \alpha_{w_m}\}$, where α_{w_i} is some $\alpha_j \in \mathcal{B}$. The *subspace based nonzero component graph*, denoted by $\Gamma_W(\mathbb{V}_\alpha) = (V, E)$, of a finite dimensional vector space with respect to W and $\mathcal{B}$ is defined as follows: $V = \mathbb{V} \setminus W$ and for $\mathbf{u}, \mathbf{v} \in V$, there is an edge between $\mathbf{u}$ and $\mathbf{v}$ if and only if either $W_{\mathbf{u}} \subset W_{\mathbf{v}}$ or $W_{\mathbf{v}} \subset W_{\mathbf{w}}$, where $\mathbf{u} = u_1\alpha_1 + u_2\alpha_2 + \cdots + u_k\alpha_k$, $1 \leq k \leq n$ and $W_{\mathbf{u}} = \{\alpha_{w_1}, \alpha_{w_2}, \ldots, \alpha_{w_m}\} \cup \{\alpha_1, \alpha_2, \ldots, \alpha_k\}$. In this paper, we determine the order, size and edge-connectivity of $\Gamma_W(\mathbb{V}_\alpha)$. We show that the graph $\Gamma_W(\mathbb{V}_\alpha)$ is Hamiltonian but not Eulerian. We also show that under some mild conditions, the graph $\Gamma_W(\mathbb{V}_\alpha)$ lies on a triangle. Finally, we determine the clique number and the chromatic number of $\Gamma_W(\mathbb{V}_\alpha)$.

Keywords: vector space, graph, maximal cliques, Hamiltonian graph.

AMS Subject Classification: 05C25, 05C69

1. Introduction

Let $G = (V(G), E(G))$ be a simple graph with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G)$. For $1 \leq i \neq j \leq n$, we write $v_i \sim v_j$ if v_i is adjacent to v_j in G. For $u \in V(G)$, $N(u)$ denotes the set of neighbours of u in G, that is, $N(u) = \{v \in V(G) : uv \in E(G)\}$, and $\deg(u) = |N(u)|$. The cardinalities of

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$V(G)$ and $E(G)$ are called the *order* and *size* of G, respectively. For any other standard graph-theoretic terminology and notation used but not explicitly recalled here, including distance, diameter, connectedness, completeness, and Hamiltonian and Eulerian graphs, we refer the reader to [6, 20].

A complete subgraph of G is called a *clique*, and a clique that is maximal with respect to inclusion is called a *maximal clique*. The *clique number* of G, written $\omega(G)$, is the order of a maximum clique in G. The *chromatic number* of G, denoted $\chi(G)$, is the minimum number of colours required to colour the vertices of G so that no two adjacent vertices receive the same colour. A subset $\alpha(G)$ of $V(G)$ is said to be *independent* if no two of its vertices are adjacent, and the *independence number* of G is the cardinality of a maximum independent set of G. A graph G is said to be *chordal* (or *triangulated*) if every cycle in G of length at least four has a chord, equivalently, G contains no induced cycle of length greater than three. The *girth* of G, denoted $gr(G)$, is the length of a shortest cycle in G, if one exists; otherwise $gr(G) = \infty$.

In 1988, Beck [1] initiated the study of graphs associated with various algebraic structures by introducing the concept of zero-divisor graphs associated to rings. Redmond [23] extended the study of zero divisor graphs of commutative rings to non-commutative rings. DeMeyer et al. [14] associated graphs to semigroups. Redmond [24] took a new approach to define zero divisor graphs with the help of an ideal of a ring. The association of a graph to a vector space has history back in 1958 by Gould [16]. Later, Chen [7] investigated on vector spaces associated with a graph. Carvalho [13] has studied vector space and the Petersen Graph. In the recent past, Manjula [18] used vector spaces and made it possible to use techniques of linear algebra in studying the graph. Intersection graphs associated with subspaces of vector spaces were studied in [3–5, 17, 25]. Recently, Das [8] introduced the graphs associated with the elements of finite dimensional vector space known as *nonzero component graph* of vector space. Das [11] showed that the graph is connected and found its domination number and independence number. He characterized the maximal cliques in the graph and found the exact clique number for some particular cases. He has also given some results on size, edge-connectivity and the chromatic number of the graph. Das [9] also introduced *subspace inclusion graph of a vector space* and studied its various graph theoretic properties like chromatic number, domination, edge-connectivity etc. More work on this can be seen in [10, 12].

In *nonzero component graph* Das [8] defined a relation that there is an edge between two non zero vectors (vertex set of a graph) if they share at least one basis vector with nonzero coefficient in their unique linear combination with respect to the basis of a vector space. In *subspace inclusion graph of a vector space*, the vertices are the non-trivial proper subspaces of $\mathbb{V}$ and there is an edge between subspaces if and only if one is contained in the other. The above two different constructions of graphs motivated us to introduce a new graph *subspace based nonzero component graph* including both elements as well as the subspace of a vector space (see [21]) and is as follows: Let $\mathbb{V}$ be a vector space over a field $\mathcal{F}$ with basis $\mathcal{B}_{\mathbb{V}} = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$. Then any vector $\mathbf{u} \in \mathbb{V}$ with unique representation as $\mathbf{u} = u_1\alpha_1 + u_2\alpha_2 + \cdots + u_n\alpha_n$, is said to have its basic representation with respect to $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$. Let W be an m dimensional

subspace of $\mathbb{V}$ with basis $\mathcal{B}_{\mathbb{W}} = \{\alpha_{w_1}, \alpha_{w_2}, \ldots, \alpha_{w_m}\}$, where α_{w_i} is some $\alpha_j \in \mathcal{B}$. We define *subspace based nonzero component graph*, denoted by $\Gamma_W(\mathbb{V}_\alpha) = (V, E)$, of a finite dimensional vector space with respect to W and $\mathcal{B}$ as follows: $V = \mathbb{V} \setminus W$ and for $\mathbf{u}, \mathbf{v} \in V$, there is an edge between $\mathbf{u}$ and $\mathbf{v}$, that is, $\mathbf{u} \sim \mathbf{v}$ or $(\mathbf{u}, \mathbf{v}) \in E$ if and only if either $W_{\mathbf{u}} \subset W_{\mathbf{v}}$ or $W_{\mathbf{v}} \subset W_{\mathbf{w}}$, where $\mathbf{u} = u_1\alpha_1 + u_2\alpha_2 + \cdots + u_k\alpha_k$, $1 \leq k \leq n$ and $W_{\mathbf{u}} = \{\alpha_{w_1}, \alpha_{w_2}, \ldots, \alpha_{w_m}\} \cup \{\alpha_1, \alpha_2, \ldots, \alpha_k\}$. In other words, $\mathbf{u} \sim \mathbf{v}$ if and only if $\mathbf{u}$ shares α_i's having nonzero coefficient in their basic representation, other than the α_i's which form the basis of W with $\mathbf{v}$ or $\mathbf{v}$ shares α_j's having nonzero coefficient in their basic representation, other than the α_j's which form the basis of W with $\mathbf{u}$. For some examples of $\Gamma_W(\mathbb{V}_\alpha)$, see Figures 1 and 2.
The paper is organized as follows. In Section 2, we determine the order, size and edge connectivity of $\Gamma_W(\mathbb{V}_\alpha)$. In addition, we find the conditions under which the graph $\Gamma_W(\mathbb{V}_\alpha)$ is triangulated. In Section 3, we show that $\Gamma_W(\mathbb{V}_\alpha)$ is Hamiltonian but not Eulerian. In Section 4, we find clique number and the chromatic number of $\Gamma_W(\mathbb{V}_\alpha)$.

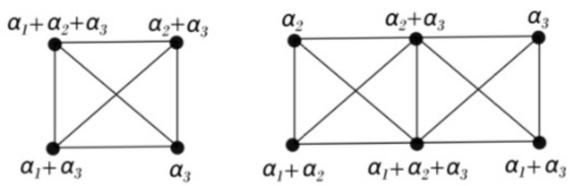

Figure 1. *$dim(\mathbb{V}) = 3$, $\mathcal{F} = \mathbb{F}_2$ and $dim(W) = 2$ and 1 respectively*

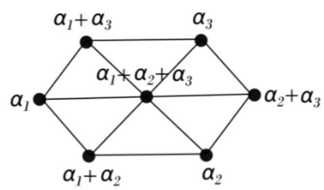

Figure 2. *$dim(\mathbb{V}) = 3$, $dim(W) = 0$, and $\mathcal{F} = \mathbb{F}_2$*

2. $\Gamma_W(\mathbb{V}_\alpha)$ is triangulated

We begin this section, by noting the fact (see [21]) that for any basis vector $\alpha_{i_j} \in \mathcal{B}_{\mathbb{V}} \setminus \mathcal{B}_{\mathbb{W}}$, we have

$$\begin{aligned} N(\alpha_{i_1} + \alpha_{i_2} + \cdots + \alpha_{i_k}) &= N(d_1\alpha_{i_1} + d_2\alpha_{i_2} + \cdots + d_k\alpha_{i_k}) \\ &= N(c_1\alpha_1 + c_2\alpha_2 + \cdots + c_m\alpha_m + d_1\alpha_{i_1} + d_2\alpha_{i_2} + \cdots + d_k\alpha_{i_k}), \end{aligned}$$

where $c_i \in \mathbb{F}$, $d_{m+1}d_{m+2}\dots d_k \neq 0$ and $d_1, d_2, \dots, d_k \in \mathbb{F}$. To determine the order and size of $\Gamma_W(\mathbb{V}_\alpha)$ and to find the minimum degree in $\Gamma_W(\mathbb{V}_\alpha)$, we require the following result on the degrees of the vertices of $\Gamma_W(\mathbb{V}_\alpha)$.

Theorem 1. [21] *Let $\mathbb{V}$ be an n-dimensional vector space over a field $\mathbb{F}_q$ and W be an m-dimensional subspace of $\mathbb{V}$. Let $\mathcal{B}_{\mathbb{V}} = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ and $\mathcal{B}_{\mathbb{W}} = \{\alpha_1, \alpha_2, \dots, \alpha_m\}$ form the basis of $\mathbb{V}$ and W, respectively. Then,*

$$deg(\sum_{j=1}^{k} d_j \alpha_{i_j}) = \left\{ \begin{array}{ll} (q-1)q^{n-1} - 1 & \textit{for one basis vector} \\ \sum_{j=1}^{k-1} \binom{k}{j}(q-1)^j q^m + (q-1)^k q^{n-k} - 1 & k \geq 2. \end{array} \right\}$$

where $\alpha_{i_j} \in \mathcal{B}_{\mathbb{V}} \setminus \mathcal{B}_{\mathbb{W}}$ and $1 \leq k \leq n-m$

Throughout this paper, the underlying field $\mathbb{F}$ is finite and $\mathbb{V}$ is finite dimensional. The following result gives the order and size of $\Gamma_W(\mathbb{V}_\alpha)$.

Theorem 2. *Let $\mathbb{V}$ be an n-dimensional vector space over a finite field $\mathcal{F}$ with q elements and W be an m-dimensional subspace of $\mathbb{V}$. Then the order of $\Gamma_W(\mathbb{V}_\alpha)$ is $q^n - q^m$ and size M of $\Gamma_W(\mathbb{V}_\alpha)$ is*

$$\frac{2q^{2m}(q^2-q+1)^{n-m} - q^{2m}(q^2-2q+2)^{n-m} - 2q^{n+m} - q^n + q^{2m} + q^m}{2}.$$

Proof. It is easy to see that the order of $\Gamma_W(\mathbb{V}_\alpha)$ is $q^n - q^m$.
Since there are $n-m$, α_i's$\in \mathcal{B}_{\mathbb{V}} \setminus \mathcal{B}_{\mathbb{W}}$, so the number of vectors having exactly one α_i with nonzero coefficient in its basic representation is $(n-m)q^m(q-1)$, i.e., $\binom{n-m}{1}q^m(q-1)$. Similarly the number of vectors having exactly two α_i's with nonzero coefficient in its basic representation is $(n-m)q^m(q-1)^2$, i.e., $\binom{n-m}{2}q^m(q-1)^2$. In general, the number of vectors having exactly k, α_i's with nonzero coefficient in its basic representation is $(n-m)q^m(q-1)^k$, i.e., $\binom{n-m}{k}q^m(q-1)^k$, where $1 \leq k \leq n-m$. By Theorem 1 and noting the fact that the sum of the degrees of all vertices in $\Gamma_W(\mathbb{V}_\alpha)$ is equal to $2M$, we have

$$\begin{aligned} 2M &= \binom{n-m}{1}q^m(q-1)\Big\{(q-1)q^{n-1} - 1\Big\} \\ &\quad + \sum_{k=2}^{n-m}\binom{n-m}{k}q^m(q-1)^k\left\{\sum_{j=1}^{k-1}\binom{k}{j}(q-1)^j q^m + (q-1)^k q^{n-k} - 1\right\} \\ &= \sum_{k=1}^{n-m}\binom{n-m}{k}q^m(q-1)^k\Big\{(q-1)^k q^{n-k} - 1\Big\} \\ &\quad + \sum_{k=2}^{n-m}\binom{n-m}{k}q^m(q-1)^k\left\{\sum_{j=1}^{k-1}\binom{k}{j}(q-1)^j q^m\right\} \end{aligned}$$

$$= \sum_{k=1}^{n-m}\binom{n-m}{k}q^m(q-1)^{2k}q^{n-k} - \sum_{k=1}^{n-m}\binom{n-m}{k}q^m(q-1)^k$$
$$+\sum_{k=2}^{n-m}\binom{n-m}{k}q^m(q-1)^k\left\{q^{k+m} - \{1+(q-1)^k\}q^m\right\}. \tag{2.1}$$

Making simplificatios, it is easy to see that

$$\sum_{k=1}^{n-m}\binom{n-m}{k}q^m(q-1)^{2k}q^{n-k} = q^{2m}(q^2-q+1)^{n-m} - q^{n+m}, \tag{2.2}$$

$$\sum_{k=1}^{n-m}\binom{n-m}{k}q^m(q-1)^k = q^n - q^m, \tag{2.3}$$

and

$$\sum_{k=2}^{n-m}\binom{n-m}{k}q^m(q-1)^k\left\{q^{k+m} - \{1+(q-1)^k\}q^m\right\}$$

$$\begin{aligned} &= q^{2m}(q^2-q+1)^{n-m} - (n-m)q^{2m+1}(q-1)\\ &\quad -q^{2m} - q^{2m}(q^2-2q+2)^{n-m} + q^{2m} + (n-m)q^{2m}(q^2-2q+1)\\ &\quad -q^{n+m} + q^{2m} + (n-m)(q-1)q^{2m}. \end{aligned} \tag{2.4}$$

Using equations (2.3) and (2.4) in equation (2.2), we have

$$\begin{aligned} 2M &= 2q^{2m}(q^2-q+1)^{n-m} - q^{2m}(q^2-2q+2)^{n-m}\\ &\quad -2q^{n+m} - q^n + q^{2m} + q^m. \end{aligned}$$

and hence the result follows. □

Theorem 3. *Let $\mathbb{V}$ be an n-dimensional vector space over a finite field $\mathcal{F}$ with q elements and W be an m-dimensional subspace of $\mathbb{V}$.*

(i) If $n = 2$ and $q = 2$, there does not exist any cycle in $\Gamma_W(\mathbb{V}_\alpha)$.

(ii) If $n = 2$ and $q \neq 2$, then every vertex of $\Gamma_W(\mathbb{V}_\alpha)$ lies on a triangle.

(iii) If $n \geq 3$, then every vertex of $\Gamma_W(\mathbb{V}_\alpha)$ lies on a triangle.

Proof. **(i)** For $n = 2$ and $q = 2$, the vertex set $\mathbf{V}$ is either $\{\alpha_1, \alpha_2, \alpha_1 + \alpha_2\}$ or $\{\alpha_2, \alpha_1 + \alpha_2\}$, corresponding to $m = 0$ and $m = 1$. Therefore, we are done.

(ii) Let $n = 2$ and $q \neq 2$, the dimension m of W is either 0 or 1. (a). Let $m = 1$, i.e., W is a hyperspace. Therefore, $\Gamma_W(\mathbb{V}_\alpha)$ is a complete graph, hence every vertex trivially lies on a triangle.
(b). Since $m = 0$ and $q \neq 2$, there exists $a \in \mathcal{F} \setminus \{0, 1\}$. For any arbitrary α_i, there exists two vertices $\alpha_i + \alpha_j$ and $a\alpha_i + a\alpha_j$, $i \neq j$ such that $\alpha_i \sim \alpha_i + \alpha_j \sim a\alpha_i + a\alpha_j \sim \alpha_i$ is a cycle of length three. Hence, every vertex of $\Gamma_W(\mathbb{V}_\alpha)$ lies on a triangle.

(iii) If $n \geq 3$, then $m \leq n - 1$. If $m = n - 1$, then $\Gamma_W(\mathbb{V}_\alpha)$ is a complete graph, hence we are done.
Now, we assume that $m < n - 1$. If $m = n - 2$, following the same arguments as is done in $(ii)b$, every vertex of $\Gamma_W(\mathbb{V}_\alpha)$ lies on a triangle. Consider $m < n-2$, our claim is that each vertex of $\Gamma_W(\mathbb{V}_\alpha)$ is a vertex of a triangle. To obtain this, we will fix a subspace W with dimension $m = k$ and $0 \leq k \leq n-3$. For any arbitrary $a \in \mathbf{V}$, such that u contains exactly s, $(s \leq n - k)$ distinct α_i's, (say, $\{\alpha_{a_1}, \alpha_{a_2}, \ldots, \alpha_{a_s}\}$) with nonzero coefficient, other than the basis elements of W in its basic representation, there always exist at least two vertices b and c with $s-1$ and $s-2$ basis vectors among the basis vectors of a, other than the basis vectors of W, in their basic representation. In other words, for any $a = \alpha_{w_1} + \alpha_{w_2} + \cdots + \alpha_{w_m} + \alpha_{a_1} + \alpha_{a_2} + \cdots + \alpha_{a_s}$, there exist $b = \alpha_{w_1} + \alpha_{w_2} + \cdots + \alpha_{w_m} + \alpha_{b_1} + \alpha_{b_2} + \cdots + \alpha_{b_{s-1}}$ and $c = \alpha_{w_1} + \alpha_{w_2} + \cdots + \alpha_{w_m} + \alpha_{c_1} + \alpha_{c_2} + \cdots + \alpha_{c_{s-2}}$, where $\{\alpha_{c_1}, \alpha_{c_2}, \ldots, \alpha_{c_{s-2}}\} \subset \{\alpha_{b_1}, \alpha_{b_2}, \ldots, \alpha_{b_{s-1}}\} \subset \{\alpha_{a_1}, \alpha_{a_2}, \ldots, \alpha_{a_s}\}$, such that $a \sim b \sim c \sim a$. So, every arbitrary vertex of $\Gamma_W(\mathbb{V}_\alpha)$ is a vertex of a triangle. $\square$

Remark 1. The property established in Theorem 3 is strictly weaker than $\Gamma_W(\mathbb{V}_\alpha)$ being chordal (triangulated). Every vertex lying on a triangle does not preclude the existence of an induced cycle of length greater than three elsewhere in the graph, so $\Gamma_W(\mathbb{V}_\alpha)$ need not be chordal, as the following example shows.

Example 1. Let $\mathbb{V}$ be a 4-dimensional vector space over $\mathcal{F} = \mathbb{F}_2$ with basis $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ and let $W = \{0\}$, so $m = 0$. Consider the four vertices

$$\alpha_1, \quad \alpha_1 + \alpha_2 + \alpha_3, \quad \alpha_2, \quad \alpha_1 + \alpha_2 + \alpha_4.$$

Since $W_{\alpha_1} = \{\alpha_1\} \subset \{\alpha_1, \alpha_2, \alpha_3\} = W_{\alpha_1+\alpha_2+\alpha_3}$, $W_{\alpha_2} = \{\alpha_2\} \subset W_{\alpha_1+\alpha_2+\alpha_3}$, $W_{\alpha_2} \subset \{\alpha_1, \alpha_2, \alpha_4\} = W_{\alpha_1+\alpha_2+\alpha_4}$, and $W_{\alpha_1} \subset W_{\alpha_1+\alpha_2+\alpha_4}$, the four vertices form a 4-cycle

$$\alpha_1 \sim \alpha_1 + \alpha_2 + \alpha_3 \sim \alpha_2 \sim \alpha_1 + \alpha_2 + \alpha_4 \sim \alpha_1.$$

Moreover $W_{\alpha_1} \not\subset W_{\alpha_2}$ and $W_{\alpha_2} \not\subset W_{\alpha_1}$, so $\alpha_1 \nsim \alpha_2$; and $W_{\alpha_1+\alpha_2+\alpha_3} \not\subset W_{\alpha_1+\alpha_2+\alpha_4}$ and $W_{\alpha_1+\alpha_2+\alpha_4} \not\subset W_{\alpha_1+\alpha_2+\alpha_3}$, so $\alpha_1 + \alpha_2 + \alpha_3 \nsim \alpha_1 + \alpha_2 + \alpha_4$. Hence this 4-cycle is induced, so $\Gamma_W(\mathbb{V}_\alpha)$ is not chordal for these parameters. Consequently, the property that $\Gamma_W(\mathbb{V}_\alpha)$ is chordal cannot hold in general, and Theorem 3 should be read strictly as a triangle-covering statement rather than a chordality result.

Now, we obtain the girth of $\Gamma_W(\mathbb{V}_\alpha)$.

Theorem 4. *Let $\mathbb{V}$ be an n-dimensional vector space over a finite field $\mathcal{F}$ with q elements and W be an m-dimensional subspace of $\mathbb{V}$. Then girth*

$$gr(\Gamma_W(\mathbb{V}_\alpha)) = \begin{cases} \infty & \text{if } n=1 \text{ and } q \leq 3 \\ \infty & n=2 \text{ and } q=2 \\ 3 & n=2 \text{ and } q\neq 2 \\ 3 & \text{if } n=1 \text{ and } q> 3 \\ 3 & \text{if } n\geq 3. \end{cases}$$

Proof. Assume that $n = 1$ and $q \leq 3$. So, there does not exist any cycle in $\Gamma_W(\mathbb{V}_\alpha)$, hence $gr(\Gamma_W(\mathbb{V}_\alpha)) = \infty$. Again, if $n = 2$ and $q = 2$, by part Theorem 3(i) , we are done.
Now, if $n = 1$ and $q > 3$, then order of $\Gamma_W(\mathbb{V}_\alpha)$ is at least 3 and every vertex a is of the form $a = c_i\alpha$, where $0 \neq c_i \in \mathcal{F}$. So, any three vertices will form a triangle. Hence, a cycle of length 3, and we are done. Finally, if either $n = 2$ and $q \neq 2$ or $n \geq 3$. The result follows from Theorem 3 (ii) and (iii). □

3. $\Gamma_W(\mathbb{V}_\alpha)$ is Hamiltonian

In this section, we first find the minimum degree and edge connectivity of $\Gamma_W(\mathbb{V}_\alpha)$. The main aim of this section is to show that the graph $\Gamma_W(\mathbb{V}_\alpha)$ is Hamiltonian but not Eulerian. To do this, we require the minimum degree of $\Gamma_W(\mathbb{V}_\alpha)$, which is obtained as follows.

Theorem 5. *Let $\mathbb{V}$ be an $n \geq 2$-dimensional vector space over a finite field $\mathcal{F}$ with q elements and W be an m-dimensional subspace of $\mathbb{V}$. Then the minimum degree δ of $\Gamma_W(\mathbb{V}_\alpha)$ is $(q-1)q^{n-1} - 1$.*

Proof. From theorem 1, the only possible degree of a vertex is either $(q-1)q^{n-1}-1$ or $\sum_{j=1}^{k-1}\binom{k}{j}(q-1)^j q^m + (q-1)^k q^{n-k} - 1$, where $2 \leq k \leq n-m$. So,

$$\delta \;=\; \min_{2\leq k\leq n-m}\left\{(q-1)q^{n-1}-1,\ \sum_{j=1}^{k-1}\binom{k}{j}(q-1)^j q^m + (q-1)^k q^{n-k} - 1\right\} \tag{3.1}$$

Our claim is to show that the sequence of vertex degrees

$$< \sum_{j=1}^{k-1}\binom{k}{j}(q-1)^j q^m + (q-1)^k q^{n-k} - 1 >$$

is monotonically increasing. Consider,

$$
\begin{aligned}
& deg(\sum_{j=1}^{s+1}\alpha_{i_j}) - deg(\sum_{j=1}^{s}\alpha_{i_j}) \\
&= \sum_{j=1}^{s}\binom{s+1}{j}(q-1)^j q^m + (q-1)^{s+1}q^{n-(s+1)} - 1 - \sum_{j=1}^{s-1}\binom{s}{j}(q-1)^j q^m - (q-1)^s q^{n-s} + 1 \\
&= \sum_{j=1}^{s}\binom{s+1}{j}(q-1)^j q^m - \sum_{j=1}^{s-1}\binom{s}{j}(q-1)^j q^m + (q-1)^{s+1}q^{n-(s+1)} - (q-1)^s q^{n-s} \\
&= \sum_{j=1}^{s}\binom{s}{j}(q-1)^j q^m + \sum_{j=1}^{s}\binom{s}{j-1}(q-1)^j q^m - \sum_{j=1}^{s-1}\binom{s}{j}(q-1)^j q^m - (q-1)^s q^{n-(s+1)} \\
&= \sum_{j=1}^{s}\binom{s}{j-1}(q-1)^j q^m + (q-1)^s q^m - (q-1)^s q^{n-(s+1)}.
\end{aligned}
$$

Since $n-m=k$ and $s \leq k-1$, we have

$$
\begin{aligned}
deg(\sum_{j=1}^{s+1}\alpha_{i_j}) - deg(\sum_{j=1}^{s}\alpha_{i_j}) &= \sum_{j=1}^{s}\binom{s}{j-1}(q-1)^j q^m + (q-1)^s q^{n-k} - (q-1)^s q^{n-(s+1)} \\
&= \sum_{j=1}^{s}\binom{s}{j-1}(q-1)^j q^m + (q-1)^s q^{n-(s+1)}(q-1) \\
&> 0.
\end{aligned}
$$

From equation (2.1) and using the fact that $n-m=k$, it is easy to see that

$$
\begin{aligned}
\delta &= \min\left\{(q-1)q^{n-1}-1,\ \sum_{j=1}^{2-1}\binom{2}{j}(q-1)^j q^m + (q-1)^2 q^{n-2} - 1\right\} \\
&= \min\left\{(q-1)q^{n-1}-1,\ 2(q-1)q^{n-k} + (q-1)^2 q^{n-2} - 1\right\}.
\end{aligned}
$$

On simplifications, we get $\delta = (q-1)q^{n-1}-1$. □

As a consequence of Theorem 5, we find the edge connectivity of $\Gamma_W(\mathbb{V}_\alpha)$. For that we need to recall the following two results on connectedness/diameter and completeness of $\Gamma_W(\mathbb{V}_\alpha)$.

Theorem 6. [21] *Let W be a subspace of a vector space $\mathbb{V}$ over a field $\mathcal{F}$. Then $\Gamma_W(\mathbb{V}_\alpha)$ is connected and $diam(\Gamma_W) = 2$, if $dim(W) < n-1$. In particular, if W is a hyperspace, then $diam(\Gamma_W) = 1$.*

Theorem 7. [21] *Let $\mathbb{V}$ be an n-dimensional vector space over a field $\mathcal{F}$ and W be a subspace of $\mathbb{V}$. Then $\Gamma_W(\mathbb{V}_\alpha)$ is complete if and only if $dim(W) = n-1$, i.e., W is a hyperspace.*

Note that if W is a hyperspace, then by Theorem 6, $\Gamma_W(\mathbb{V}_\alpha)$ is complete. Hence degree of every vertex is same. Therefore edge connectivity of $\Gamma_W(\mathbb{V}_\alpha)$ is $(q-1)q^{n-1}-1$. On the other hand if W is not a hyperspace, i.e., $dim(W) < n-1$, then $diam(\Gamma_W) = 2$, by Theorem 7. Therefore, its edge connectivity is equal to its minimum degree (see [22],). i.e., $(q-1)q^{n-1}-1$.
Now, we are in a position to determine Hamiltonicity of $\Gamma_W(\mathbb{V}_\alpha)$, when the base field $\mathcal{F}$ is of dimension greater or equal to 2. The following result will be used in the sequel.

Theorem 8. [15]. *If G is a connected graph with minimum degree δ such that $\delta \geq \frac{|G|}{2}$, then G is Hamiltonian.*

Theorem 9. *Let $\mathbb{V}$ be an $n > 1$-dimensional vector space over a finite field $\mathcal{F}$ with $q > 2$ and W be an m-dimensional subspace of $\mathbb{V}$. Then $\Gamma_W(\mathbb{V}_\alpha)$ is Hamiltonian.*

Proof. In order to prove the result, we claim that $q^n > 2q^{n-1}+1$. Since $n > 1$ and $q > 2$, we have $q^{n-1} \geq q \geq 3$ and $q-2 \geq 1$. Therefore, $q^{n-1}(q-2) \geq 3 > 1$, which gives $q^n - 2q^{n-1} > 1$, i.e., $q^n > 2q^{n-1}+1$.
Now, for $n \geq 2$, $q^n > 2q^{n-1}+1$ implies that $2q^n - 2q^{n-1} - 2 > q^n - 1$, i.e., $q^n - q^{n-1} - 1 > \frac{q^n-1}{2} > \frac{q^n-q^m}{2}$. Using Theorems 2 and 5, we have $\delta > \frac{|\mathbf{V}|}{2}$. Therefore, by Theorem 8, $\Gamma_W(\mathbb{V}_\alpha)$ is Hamiltonian. □

Theorem 9 gives information about the Hamiltonicity of the graph $\Gamma_W(\mathbb{V}_\alpha)$, only when $\mathcal{F}$ has more than two elements. However this theorem provides insufficient information about the Hamiltonicity of the graph $\Gamma_W(\mathbb{V}_\alpha)$, when the base field has exactly two elements, that is, when $\mathcal{F} = \mathbb{F}_2$. To overcome this problem, we make use of the independence number of the graph and the classical theorem on sufficient conditions of Hamiltonicity of a graph.

Theorem 10. [21] *Let $\mathbb{V}$ be an n-dimensional vector space over a field $\mathcal{F}$ and W be an m-dimensional subspace of $\mathbb{V}$. Then the independence number of $\Gamma_W(\mathbb{V}_\alpha)$ is $dim(\mathbb{V}) - dim(W)$.*

Theorem 11. [19] *If G is a 2-connected graph with minimum degree δ and independence number $\alpha(G)$ such that $\delta \geq max\{\frac{|G|+2}{3}, \alpha(G)\}$, then G is Hamiltonian.*

Now, we prove the main result of this section.

Theorem 12. *Let $\mathbb{V}$ be an $n \geq 3$-dimensional vector space over a finite field $\mathcal{F}$ with $q = 2$ and W be an m-dimensional subspace of $\mathbb{V}$, then $\Gamma_W(\mathbb{V}_\alpha)$ is Hamiltonian.*

Proof. To prove that $\Gamma_W(\mathbb{V}_\alpha)$ is Hamiltonian, first we claim that $\delta \geq max\{\frac{|\mathbf{V}|+2}{3}, \alpha(\Gamma_W)\}$, where $\alpha(\Gamma_W)$ is an independence number of $\Gamma_W(\mathbb{V}_\alpha)$. Since $n \geq 3$ and $0 \geq m \leq n-1$, it is easy to see that $2^{n-1} + 2^m - 5 \geq 0$ which implies that $(2^n - 2) + (2^{n-1} - 1) \geq 2^n - 2^m + 2$, which implies that $3(2^{n-1} - 1) \geq 2^n - 2^m + 2$.

Hence $(2^{n-1}-1) \geq \frac{2^n-2^m+2}{3}$. So, for $q=2$, $\delta = 2^{n-1}-1$ and $|\mathbf{V}| = 2^n - 2^m$, the following inequality holds

$$\delta \geq \frac{|\mathbf{V}|+2}{3}.$$

Also, by Theorem 10, $\alpha(\Gamma_W) = n-m$. For $n \geq 3$, we have

$$\delta = 2^{n-1} - 1 \geq n - m.$$

Combining the above inequalities, the claim is proved.
Now, it suffices to show that $\Gamma_W(\mathbb{V}_\alpha)$ is 2-connected. Consider the following three cases.
(a). If $m = n-1$, then by Theorem 7, $\Gamma_W(\mathbb{V}_\alpha)$ is a complete graph of $2^{n-1} > 3$ vertices and so Hamiltonian.
(b). Suppose $m = n-2$, we will show that removal of any one vertex does not make $\Gamma_W(\mathbb{V}_\alpha)$ disconnected, i.e., $\Gamma_W(\mathbb{V}_\alpha) - \beta$ is connected for any nonzero vector $\beta \in \mathbf{V}$. Let $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ be a basis of $\mathbb{V}$. Without loss of generality, assume that $\{\alpha_1, \alpha_2, \ldots, \alpha_{n-2}\}$ is the basis of subspace W. Let $u \in \mathbf{V}$. Then u is of the form $u = \sum_1^{n-2} c_i\alpha_i + d_{n-1}\alpha_{n-1} + d_n\alpha_n$, where $c_i \in \mathcal{F}$ and $d_1 \neq 0 \neq d_2 \in \mathcal{F}$. Consider two arbitrary nonzero vectors u and v other than β in $\mathbb{V}$. If they are adjacent in $\Gamma_W(\mathbb{V}_\alpha)$, we are done. If they are not adjacent, then $u = \sum_1^{n-2} c_i\alpha_i + \alpha_{n-1}$ and $v = \sum_1^{n-2} c_i\alpha_i + \alpha_n$. Consider $w = u + v$. Clearly, $u \sim w$ and $v \sim w$ in $\Gamma_W(\mathbb{V}_\alpha)$. So, there is a path between u and v. But, if w is the removed vertex β, then this path between u and v does not exist in $\Gamma_W(\mathbb{V}_\alpha) - \beta$. However, there exists another vertex x different from w, which is free from the basis elements of W, i.e., $x = \alpha_{n-1} + \alpha_n$ such that $u \sim x$ and $v \sim x$ in $\Gamma_W(\mathbb{V}_\alpha) - \beta$. Hence $\Gamma_W(\mathbb{V}_\alpha) - \beta$ is connected and thereby $\Gamma_W(\mathbb{V}_\alpha)$ is 2-connected.
(c). If $m < n-2$, we will follow the same procedure and prove that removal of any one vertex does not make $\Gamma_W(\mathbb{V}_\alpha)$ disconnected, i.e., $\Gamma_W(\mathbb{V}_\alpha) - \beta$ is connected for any nonzero vector $\beta \in \mathbf{V}$. Let $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ be a basis of $\mathbb{V}$. Without loss of generality, assume that $\{\alpha_1, \alpha_2, \ldots, \alpha_{n-3}\}$ is the basis of subspace W. Let u and v be two arbitrary nonzero vectors other than β in $\mathbb{V}$. If they are adjacent in $\Gamma_W(\mathbb{V}_\alpha)$, we are done. If not, since $u, v \neq 0$, there exist α_i, α_j other than the basis vectors of W, which have nonzero coefficient in the basic representation of u and v, respectively. Moreover, as u and v are not adjacent, $\alpha_i \neq \alpha_j$. Consider $w = \alpha_i + \alpha_j$. Then, $u \sim w$ and $v \sim w$ in $\Gamma_W(\mathbb{V}_\alpha)$. This provides a path between u and v. However, if w is the removed vertex β, then this path between u and v does not exist in $\Gamma_W(\mathbb{V}_\alpha) - \beta$. But as $n \geq 3$ and $m \leq n-3$, there exists α_k in the mentioned basis other than α_i, α_j. Then consider the vertex $x = \alpha_i + \alpha_j + \alpha_k$ and observe that $u \sim x$ and $v \sim x$ in $\Gamma_W(\mathbb{V}_\alpha) - \beta$. Hence $\Gamma_W(\mathbb{V}_\alpha) - \beta$ is connected and thereby $\Gamma_W(\mathbb{V}_\alpha)$ is 2-connected.
Since, $\delta \geq max\{\frac{|\mathbf{V}|+2}{3}, \alpha(\Gamma_W)\}$ and $\Gamma_W(\mathbb{V}_\alpha)$ is 2-connected, therefore, by Theorem 11, $\Gamma_W(\mathbb{V}_\alpha)$ is Hamiltonian. □

Remark 2. For $n = 1$ and $q > 3$, $\Gamma_W(\mathbb{V}_\alpha)$ is complete and hence it is Hamiltonian.

Now, we show that the graph $\Gamma_W(\mathbb{V}_\alpha)$ is not Eulerian.

Theorem 13. *Let $\mathbb{V}$ be an n-dimensional vector space over a field $\mathcal{F}$ and W be an m-dimensional subspace of $\mathbb{V}$. Then $\Gamma_W(\mathbb{V}_\alpha)$ is not Eulerian.*

Proof. From Theorem 1, it is easy to see that the dimension of subspace W has no effect on whether the graph is Eulerian or not.
If $q = 2$, by Theorem 1, every vertex $u = \sum_{j=1}^{k} d_j \alpha_{i_j}$ is of odd degree. Again, if q is odd prime, $\Gamma_W(\mathbb{V}_\alpha)$ is not Eulerian, since, in this case degree of every vertex is odd. Thus the graph is not Eulerian in any case. □

4. Maximal cliques in $\Gamma_W(\mathbb{V}_\alpha)$

In this section, we find the clique number of $\Gamma_W(\mathbb{V}_\alpha)$. For $\beta \in \Gamma_W(\mathbb{V}_\alpha)$, let W_β be the set of α_i's with nonzero coefficients in the basic representation of β with respect to $\{\alpha_1, \alpha_2, \ldots \alpha_n\}$. It is to be noted that $W_{\alpha_1+\alpha_2+\ldots\alpha_j} \subset W_{\alpha_1+\alpha_2\cdots+\alpha_{j+1}}$ for $1 \leq j \leq n-1$. Moreover, two distinct β's may have same W_β . Also $m+1 \leq |W_\beta| \leq n$, for all $\beta \in \Gamma_W(\mathbb{V}_\alpha)$.

Theorem 14. *Let $\mathbb{V}$ be an n-dimensional vector space over a finite field $\mathcal{F}$ with q elements and W be an m-dimensional subspace of $\mathbb{V}$. Then clique number $\omega(\Gamma_W(\mathbb{V}_\alpha))$ is $\sum_{i=1}^{n-m} q^m(q-1)^i$.*

Proof. Let $\mathcal{B}_\mathbb{V} \setminus \mathcal{B}_\mathbb{W} = \{\alpha_{1'}, \alpha_{2'}, \ldots, \alpha_{(n-m)'}\}$ denote the $n-m$ basis vectors of $\mathbb{V}$ not belonging to W. For $\beta \in V$, recall that W_β is the union of the basis of W with the support of β among $\{\alpha_{1'}, \ldots, \alpha_{(n-m)'}\}$, and two vertices $u, v \in V$ are adjacent if and only if W_u and W_v are comparable under inclusion. Consequently, for any clique $\mathbf{M}$ of $\Gamma_W(\mathbb{V}_\alpha)$, the family $\{W_\beta : \beta \in \mathbf{M}\}$ is a chain (totally ordered by inclusion) in the lattice of subsets of $\mathcal{B}_\mathbb{V}$ containing the basis of W. Writing $T_\beta = W_\beta \setminus \mathcal{B}_\mathbb{W}$ for the "extra" support of β, this is equivalent to $\{T_\beta : \beta \in \mathbf{M}\}$ being a chain of nonempty subsets of the $(n-m)$-element set $\{\alpha_{1'}, \ldots, \alpha_{(n-m)'}\}$.
Since a chain of nonempty subsets of an $(n-m)$-element set has length at most $n-m$, the distinct values taken by T_β over $\mathbf{M}$ form a chain $T_1 \subsetneq T_2 \subsetneq \cdots \subsetneq T_r$ with $r \leq n-m$; since each step of a strict chain of subsets adds at least one element, $|T_i| \geq i$ for $1 \leq i \leq r$. For a fixed value T_i, the vertices β with $T_\beta = T_i$ are pairwise adjacent (their W_β's coincide, hence are comparable), and there are exactly $q^m(q-1)^{|T_i|}$ such vertices (any of the q^m choices on the coordinates of W, and a nonzero coefficient, $q-1$ choices, on each of the $|T_i|$ coordinates in T_i). Hence

$$|\mathbf{M}| = \sum_{i=1}^{r} q^m(q-1)^{|T_i|}.$$

Since $q > 1$, the function $i \mapsto q^m(q-1)^i$ is strictly increasing, so $|\mathbf{M}|$ is maximized by taking $r = n - m$ and $|T_i| = i$ for each i, i.e., by taking $T_1 \subsetneq T_2 \subsetneq \cdots \subsetneq T_{n-m}$ to be a *maximal* chain of nonempty subsets of $\{\alpha_{1'}, \ldots, \alpha_{(n-m)'}\}$, in which case

$$|\mathbf{M}| = \sum_{i=1}^{n-m} q^m(q-1)^i.$$

Since $\mathbf{M}$ was an arbitrary clique of $\Gamma_W(\mathbb{V}_\alpha)$, this shows $|\mathbf{M}| \leq \sum_{i=1}^{n-m} q^m(q-1)^i$ for *every* clique $\mathbf{M}$ in the graph, so this bound applies to the clique number itself:

$$\omega(\Gamma_W(\mathbb{V}_\alpha)) \leq \sum_{i=1}^{n-m} q^m(q-1)^i.$$

It remains to observe that such a clique exists and is maximal. Fix a maximal chain, say $T_i = \{\alpha_{1'}, \alpha_{2'}, \ldots, \alpha_{i'}\}$ for $1 \leq i' \leq n-m$, and let $\mathbf{M}$ consist of all $\beta \in V$ with $T_\beta \in \{T_1, \ldots, T_{n-m}\}$. Any two vertices of $\mathbf{M}$ have comparable W_β's (either equal, if they share the same T_i, or nested, since the T_i's are nested), so $\mathbf{M}$ is a clique, of size $\sum_{i=1}^{n-m} q^m(q-1)^i$ by the count above. If $\mathbf{M}$ were not maximal, some $v \notin \mathbf{M}$ would be adjacent to every vertex of $\mathbf{M}$, forcing T_v to be comparable to every T_i, $1 \leq i \leq n-m$; but $T_1, \ldots, T_{n-m}$ already form a maximal chain of nonempty subsets of an $(n-m)$-element set, so no further nonempty subset $T_v \notin \{T_1, \ldots, T_{n-m}\}$ can be comparable to all of them. Hence $\mathbf{M}$ is a maximal clique of size $\sum_{i=1}^{n-m} q^m(q-1)^i$. Combined with the upper bound established above, this shows the clique number of $\Gamma_W(\mathbb{V}_\alpha)$ over the whole graph equals $\sum_{i=1}^{n-m} q^m(q-1)^i$, attained by the clique $\mathbf{M}$ constructed above:

$$\omega(\Gamma_W(\mathbb{V}_\alpha)) = \sum_{i=1}^{n-m} q^m(q-1)^i.$$

□

Remark 3. A maximal clique $\mathbf{M}$ contains at most one α, because if $\mathbf{M}$ contain α_i and α_j, then $\alpha_i \nsim \alpha_j$, which contradicts that $\mathbf{M}$ is a clique.

Corollary 1. *If $\mathcal{F} = \mathbb{F}_2$, then clique number $\omega(\Gamma_W(\mathbb{V}_\alpha)) = (n-m)2^m$.*

Corollary 2. *If $\mathcal{F} = \mathbb{F}_2$, and $\chi(\Gamma_W(\mathbb{V}_\alpha))$ be the chromatic number of $\Gamma_W(\mathbb{V}_\alpha)$, then*

$$(n-m)2^m \leq \chi(\Gamma_W(\mathbb{V}_\alpha)) \leq \frac{(n-m-1)(2^m-1)+2^n}{2}.$$

Proof. For any graph G, $\omega(G) \leq \chi(G)$. Therefore, the lower bound follows from Corollary 1. For the upper bound, we use the following result of [2]

$$\chi(G) \leq \frac{\omega(G) + |V(G)| + 1 - \alpha(G)}{2},$$

where $\alpha(G)$ is the independence number of G. Using Corollary 1 and Theorems 2 and 10, we have $\omega(\Gamma_W(\mathbb{V}_\alpha)) = (n-m)2^m$, $|V(\Gamma_W(\mathbb{V}_\alpha))| = 2^n - 2^m$, and $\alpha(\Gamma_W(\mathbb{V}_\alpha)) = n-m$. Substituting,

$$\begin{aligned}
\chi(\Gamma_W(\mathbb{V}_\alpha)) &\leq \frac{(n-m)2^m + (2^n - 2^m) + 1 - (n-m)}{2} \\
&= \frac{(n-m)(2^m-1) - (2^m-1) + 2^n}{2} \\
&= \frac{(n-m-1)(2^m-1) + 2^n}{2}.
\end{aligned}$$

□

5. Conclusion

In this present work, we studied subspace based nonzero component graph $\Gamma_W(\mathbb{V}_\alpha)$ on a finite dimensional vector space $\mathbb{V}$. We have shown that for $n = 3$ and $q \geq 2$, every vertex of $\Gamma_W(\mathbb{V}_\alpha)$ lies on a triangle. We also provide the exact value of minimum degree and edge connectivity of $\Gamma_W(\mathbb{V}_\alpha)$. For $n \geq 3$ and $q = 2$, we have shown that $\Gamma_W(\mathbb{V}_\alpha)$ is Hamiltonian. Moreover, $\Gamma_W(\mathbb{V}_\alpha)$ is not Eulerian. We also find the clique number and the chromatic number of $\Gamma_W(\mathbb{V}_\alpha)$. For future research, we can think of investigating the following. (i) When $\Gamma_W(\mathbb{V}_\alpha)$ is regular, (ii) whether $\Gamma_W(\mathbb{V}_\alpha)$ is a line graph, (iii) characterize line graphs of $\Gamma_W(\mathbb{V}_\alpha)$, (iv) to find genus of $\Gamma_W(\mathbb{V}_\alpha)$.

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