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Short Note

Bounds of point-set domination number

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Abstract: A subset D of the vertex set $V(G)$ in a graph G is a point-set dominating set (or, in short, psd-set) of G if for every set $S \subseteq V - D$, there exists a vertex $v \in D$ such that the induced subgraph $\langle S \cup \{v\} \rangle$ is connected. The minimum cardinality of a psd-set of G is called the point-set domination number of G. In this paper, we establish two sharp lower bounds for point-set domination number of a graph in terms of its diameter and girth. We characterize graphs for which lower bound of point set domination number is attained in terms of its diameter. We also establish an upper bound and give some classes of graphs which attains the upper bound of point set domination number.

Keywords: Domination, Point-set Domination, Domination number

AMS Subject classification: 05C69

1. Introduction

For standard terminologies used in this paper we refer to books by F. Harary [8] and Chartrand [2]. Throughout this paper we consider simple, finite, undirected and connected graphs. For any graph G, the set $V(G)$ (or, simply V) and $E(G)$ (or, simply E) represents its vertex set and edge set respectively.
The *neighborhood of a vertex* v in a graph G, denoted by $N_G(v)$ (or simply, $N(v)$), is the set of all vertices in G adjacent with the vertex v. The set $N_G(v) \cup \{v\}$ is the

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closed neighborhood of vertex v in G and is denoted by $N_G[v]$ (or simply, $N[v]$). For any set $S \subset V$, $N(S) = \cup_{v \in S} N[v]$. The maximum degree among the vertices of G is denoted by $\Delta(G)$.The *diameter* of G is given by $\text{diam}(G) = \max\{d(u,v): \ u,v \in V\}$ where $d(u,v)$ denotes distance between u and v. A spider graph is a tree with at most one vertex of degree greater than 2. In other words, a spider graph is a tree homeomorphic to star $K_{1,n}$ for some $n \geq 1$.

A subset D of the vertex set $V(G)$ of a graph G is called a dominating set of G if every vertex not in D is adjacent to a vertex in D. The minimum cardinality of a dominating set of G is called the domination number of G and is denoted by $\gamma(G)$. The study of domination and its variants is one of the major research areas within graph theory. The two books by Haynes et al. [9, 10] provide a comprehensive treatment of the fundamental concepts and surveys on several advanced topics.

The following theorem provides bounds for the domination number in terms of the order of the graph and the maximum degree $\Delta(G)$.

Theorem 1. [10] *Let G be any graph of order n and maximum degree $\Delta(G)$. Then*

$$\frac{n}{1+\Delta(G)} \leq \gamma(G) \leq \left\lfloor \frac{n}{2} \right\rfloor. \tag{1}$$

Since the introduction of domination, more than 85 variants of concept of domination have came into existence [9, 10] and have been extensively studied. One such concept is point set domination in graphs. Point-set domination as a concept was introduced by Sampathkumar and Pushpa Latha [12] in 1993 purely from theoretical interest by generalizing the notion of domination. Due to its applicability, point-set domination has seen stupendous development and several variants of point-set domination [3–6, 11] like global point-set domination, 2-point set domination, point-set tree domination etc., have also been introduced over the years.

A subset D of the vertex set $V(G)$ in a graph G is a point-set dominating set (or, in short, psd-set) of G if for every set $S \subseteq V - D$, there exists a vertex $v \in D$ such that the induced subgraph $\langle S \cup \{v\} \rangle$ is connected. The minimum cardinality of a psd-set of G is called the point-set domination number of G and is denoted by $\gamma_p(G)$. The point-set domination number was introduced by Sampathkumar and Pushpa Latha in [12].

This definition can be seen as a natural extension of the concept of domination (cf. [9, 10]) by using the interpretation that a subset D of the vertex set V of G is a dominating set if and only if for every singleton subset $\{s\}$ of $V - D$, there exists a vertex d in D such that the induced subgraph $\langle \{s\} \cup \{d\} \rangle$ is connected.

The condition for psd-set reduces to usual domination if $S = \{s\}$ and hence every psd-set is a dominating set. Thus

$$\gamma(G) \leq \gamma_p(G). \tag{2}$$

Proposition 1. [12] *If D is a psd-set of a graph G, then $d(u,v) \leq 2$ for all $u, v \in V - D$. Also, if G is a graph with maximum degree $\Delta(G)$ and order n, then*

$$\gamma_p(G) \leq n - \Delta(G). \tag{3}$$

Inequalities (1), (2) and (3) lead to the inequality

$$\frac{n}{1+\Delta(G)} \leq \gamma_p(G) \leq n - \Delta(G). \tag{4}$$

In [7], the characterization was given for the extremal graphs for the lower bound given in (4) for point-set domination number.

Theorem 2. *[7] Let G be any graph of order n and maximum degree $\Delta(G)$. Then $\gamma_p(G) = \frac{n}{1+\Delta(G)}$ if and only if $\Delta(G) = n-1$.*

In this paper, we first establish another lower bound for point-set domination number of a graph in terms of its diameter and thereafter, we characterize extremal graphs for the lower bound. Further, though characterizing extremal graphs for the upper bound given in (4) for point-set domination number is a complex problem, we provide some classes of extremal graphs for the upper bound.

The following useful observations made in [1, 12] are easy consequences of the definition of psd-sets.

Proposition 2. [1] *Let G be any graph. Then, $D \subseteq V$ is a psd-set of G if and only if every independent subset W in $V - D$ is contained in $N(u)$ for some $u \in D$.*

Theorem 3. *[12] For any tree T of order n and maximum degree Δ, $\gamma_p(T) = n - \Delta$.*

Here we list psd number of some well known classes of graphs.

1. $\gamma_p(P_n) = n-2; n \geq 3$.
2. $\gamma_p(C_n) = n-2; n \geq 6$.
3. $\gamma_p(K_n) = 1$.
4. $\gamma_p(K_{r,s}) = 2; r \geq 2, s \geq 2$.

Note that if $G \in \{P_n, C_n\}$, then $\frac{n}{1+\Delta(G)} = \frac{n}{3}$, $\gamma(G) = \lceil \frac{n}{3} \rceil$, $\gamma_p(G) = n - \Delta(G)$. Therefore $\frac{n}{1+\Delta(G)} < \gamma(G) < \gamma_p(G) = n - \Delta(G)$. Also, if $G = K_n$, $\frac{n}{1+\Delta(G)} = \gamma(G) = \gamma_p(G) = n - \Delta(G) = 1$. In $K_{r,s}$, $\frac{n}{1+\Delta(G)} = \frac{r+s}{1+\max\{r,s\}}$, $\gamma(G) = \gamma_p(G) = 2$ $n - \Delta(G) = r + s - \max\{r, s\}$, therefore $\frac{n}{1+\Delta(G)} < \gamma(G) = \gamma_p(G) < n - \Delta(G)$.

2. Bounds on Point-set Domination Number

In the next theorem, we establish a lower bound for point-set domination number of a graph in terms of its diameter.

Theorem 4. *For any graph G, $\gamma_p(G) \geq \mathrm{diam}(G) - 1$.*

Proof. Let $\mathrm{diam}(G) = k$ and P be any diametrical path of G. Then $|V(P)| = k+1$. We claim that $\gamma_p(G) \geq k-1$. Let D be any psd-set of G. From Proposition 1, distance between any two vertices of $V(G) - D$ is at most 2 and consequently, $|(V-D) \cap V(P)| \leq 3$.
Hence $\gamma_p(G) \geq (k+1) - |(V-D) \cap V(P)|$. Now if $|(V-D) \cap V(P)| \leq 2$, then $\gamma_p(G) \geq k-1$ and we are through in this case. If $|(V-D) \cap V(P)| = 3$, let $(V-D) \cap V(P) = \{x, y, z\}$. Let $w \in D$ such that w is adjacent to y. Clearly $w \in D - (V(G) - V(P))$ and hence $\gamma_p(G) \geq |D \cap V(P)| + 1 = k-1$. Thus $\gamma_p(G) \geq k-1$ □

As an immediate consequence of Theorem 4 and the bounds for point-set domination number for a graph given in equation (4), we have the following result.

Corollary 1. *For any graph G,* $\max\left\{\frac{n}{1+\Delta(G)}, \mathrm{diam}(G) - 1\right\} \leq \gamma_p(G) \leq n - \Delta(G)$.

Proof. The results follows from (4) and Theorem 4. □

Our immediate aim is to characterize extremal graphs for both lower and upper bound for point-set domination number obtained in above theorem. In this direction, we first introduce some definitions and notations.

Definition 1. Consider two paths P_l and P_m of length l and m respectively, and a complete bipartite graph $K_{p,q}$ with bi-partition (V_1, V_2) such that $|V_1| = p$ and $|V_2| = q$. The graph obtained by joining all vertices of V_1 to an end vertex u of path P_l and all vertices of V_2 to an end vertex v of a path P_m is denoted by $K_{p,q}^{l,m}$.

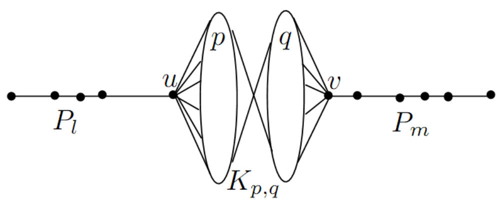

Figure 1. **Graph** $K_{p,q}^{l,m}$.

Clearly, the vertex set of graph $K_{p,q}^{l,m}$ can be partitioned into 4 sets V_1, V_2, V_3 and V_4 of cardinalities p, q, l and m, respectively such that $\langle V_1 \cup V_2 \rangle \cong K_{p,q}$, $\langle V_3 \rangle \cong P_l$ and $\langle V_4 \rangle \cong P_m$.

Theorem 5. *For a graph G of order n,* $\gamma_p(G) = \max\left\{\frac{n}{1+\Delta(G)}, \mathrm{diam}(G) - 1\right\}$ *if and only if one of the following holds:*

i. G has a spanning tree isomorphic to star $K_{1,n-1}$,

ii. *G has a spanning tree isomorphic to a spider graph having at most two legs of length greater than* 1*, (say)* P_1 *and* P_2 *with* $l(P_1) + l(P_2) = \text{diam}(G)$*,*

iii. *G has a spanning subgraph H isomorphic to graph* $K_{p,q}^{l,m}$ *with* $l + m = \text{diam}(G) - 1$*.*

Proof. If $\gamma_p(G) = \frac{n}{1+\Delta(G)}$, then by Theorem 2, (i) holds. Suppose $\gamma_p(G) = k - 1$, where $k = \text{diam}(G)$. Let D be a $\gamma_p(G)$-set of G. It follows from proof of Theorem 4 that $|(V - D) \cap V(P)| = 2$ or 3.

If $|(V - D) \cap V(P)| = 3$, then $|D \cap V(P)| = k - 2$. Since $|D| = k - 1$, therefore $D = (D \cap V(P)) \cup \{w\}$ for some $w \in V(G) - V(P)$. Further using the fact that D is a psd-set of G and P is a diametrical path of G, it can be easily seen that $V(G) - D \subseteq N(w)$ and thus (ii) holds.
If $|(V - D) \cap V(P)| = 2$, then let $(V - D) \cap V(P) = \{x, y\}$. In view of argument given in the previous case, $d(x, y) = 2$ gives (ii). If $d(x, y) = 1$, then again as D is a psd-set of G and P is a diametrical path of G, it can be proved that $V - D \subseteq N(x_1) \cup N(y_1)$ where $\{x_1\} = D \cap N(x)$ and $\{y_1\} = D \cap N(y)$. Furthermore since P is a diametrical path of G, $N(x_1)$ and $N(y_1)$ are disjoint subsets of $V(G)$ and every vertex of $N(x_1)$ is adjacent to each vertex of $N(y_1)$. Whence condition (iii) holds.
Conversely, suppose G satisfies condition (i), then $\Delta(G) = n - 1$ and $\text{diam}(G) = 2$. By Theorem 2, $\gamma_p(G) = \frac{n}{1+\Delta(G)} = \max\{\frac{n}{1+\Delta(G)}, \text{diam}(G) - 1\}$. Now suppose G satisfies (ii) and $\Delta(G) \neq n - 1$. Let w be the central vertex of G. Then $V(G) - N(w)$ will form a psd-set of G with cardinality $k - 1 = \text{diam}(G) - 1$. By Theorem 4 $\gamma_p(G) = \text{diam}(G) - 1$. Next suppose G satisfies condition (iii), then $V_3 \cup V_4$ forms a psd-set of G and therefore $\gamma_p(G) \leq |V_3 \cup V_4| = \text{diam}(G) - 1$. By Theorem 4, we conclude $\gamma_p(G) = \text{diam}(G) - 1$. Hence the result. □

After having characterized extremal graphs for the lower bounds of point-set domination number, next we focus on the upper bound for the point-set domination number of a graph given in equation (4). First we observe that the upper bound in equation (4) is a sharp bound for the point-set domination number of a graph. In fact, for any tree T, $\gamma_p(T) = n - \Delta(T)$ (Theorem 3). However, for a graph G, the difference $(n - \Delta(G)) - \gamma_p(G)$ could be made arbitrarily large. For example, if we consider the complete bipartite graph $K_{r,s}, r > s$, $\gamma_p(G) = 2$ and $(n - \Delta(G)) - \gamma_p(G) = s - 2$ which depends on s and hence could be made arbitrarily large.

Characterizing graphs whose point set domination number attains the upper bound appears to be a complex problem. However, in the next section, we make an attempt to explore extremal graphs for the upper bound and in the process we identify some classes of graphs attaining the upper bound.

3. Some classes with $\gamma_p(G) = |V(G)| - \Delta(G)$

A graph in which the vertex set $V(G)$ can be partitioned into two subsets Ω and I such that $\langle\Omega\rangle$ is a clique and I is an independent set in G is called a split graph G with partition (Ω, I). It is easy to observe that for any split graph G with partition (Ω, I), $\Delta(G) \geq |\Omega|$.

Theorem 6. *If G is a split graph, then $\gamma_p(G) = |V(G)| - \Delta(G)$.*

Proof. Let G be a split graph of order n and $V(G) = \Omega \cup I$, where Ω is a clique in G and I is an independent subset of G. We will prove that $\gamma_p(G) = n - \Delta(G)$. Let D be any $\gamma_p(G)$-set. We have two possibilities.

Case 1. $(V - D) \cap I \neq \emptyset$
Let $S = (V - D) \cap I$. Since D is a psd-set, there exists some $w \in \Omega$ such that $S \subseteq N(w)$. We claim that $V - D \subseteq N(w)$. Consider

$$\begin{aligned} V - D &= ((V - D) \cap I) \cup ((V - D) \cap (\Omega - \{w\})) \\ &= S \cup ((V - D) \cap (\Omega - \{w\})) \\ &\subseteq N(w). \end{aligned}$$

Then $\gamma_p(G) = |D| \geq n - |V - D| \geq n - |N(w)| \geq n - \Delta(G)$.

Case 2. $(V - D) \cap I = \emptyset$.
Then $V - D \subseteq \Omega$. Again, $\gamma_p(G) = |D| \geq n - |V - D| \geq n - |\Omega| \geq n - \Delta(G)$.
Thus in both cases, we obtain that $\gamma_p(G) \geq n - \Delta(G)$. From inequality (4) it follows that $\gamma_p(G) = n - \Delta(G)$. □

The Cartesian product of G_1 and G_2, denoted as $G_1 \Box G_2$, has $V(G_1) \Box V(G_2)$ as vertex set and two vertices (u_1, u_2) and (v_1, v_2) of $G_1 \Box G_2$ are adjacent if either $u_1 = v_1$ and $u_2 v_2 \in E(G_2)$ or $u_1 v_1 \in E(G_1)$ and $u_2 = v_2$.
Cartesian product of paths P_n and P_m is called grid and is denoted by $P_n \Box P_m$.

Theorem 7. *For the graph $G = C_n \Box K_2$ $(n \neq 3)$, $\gamma_p(G) = 2n - 3$.*

Proof. For the graph $G, |V(G)| = 2n$ and $\Delta(G) = 3$ and therefore by inequality (4), $\gamma_p(G) \leq 2n - 3$.
Let D be a $\gamma_p(G)$ set. This implies $diam(\langle V - D\rangle) \leq 2$. But the only induced subgraph of diameter 2 in this graph is isomorphic to either $K_{1,3}$ or C_4 which means $|V - D| \leq 4$.
If $|V - D| = 4$, then $V - D$ is isomorphic to one of C_4 and $K_{1,3}$. But in either case, D ceases to be a psd-set of G. Hence it follows that $|V - D| \leq 3$, that is, $|D| \geq 2n - 3$.
Hence the result. □

Theorem 8. *For the graph $G = C_n \Box P_m$, $(n, m \geq 3)$, $\gamma_p(G) = nm - 4$.*

Proof. For the graph $G, |V(G)| = nm$ and $\Delta(G) = 4$ and therefore by inequality (4), $\gamma_p(G) \leq nm - 4$.
Let D be a $\gamma_p(G)$-set, which implies $diam(\langle V - D \rangle) \leq 2$. The maximum number of vertices possible in an induced subgraph of G with diameter at most 2 is 5. Thus $|V - D| \leq 5$.
In case $|V - D| = 5$, $V - D$ is isomorphic to $K_{1,4}$. But, then, D ceases to be a psd-set of G. Hence $|V - D| \leq 4$, and the result follows. □

Theorem 9. *For a graph $G = C_r \square C_s$, $\gamma_p(G) = n - \Delta(G)$, where $n = rs$.*

Proof. Follows immediately on lines similar to the proof of Theorem 8. □

Theorem 10. *For a grid $G = P_n \square P_m, (n, m \geq 3)$ $\gamma_p(G) = nm - 4$.*

Proof. Follows immediately on lines similar to the proof of Theorem 8. □

In the next Theorem we prove that middle graph of cycle graph also attains the upper bound of psd number in the inequality (3).

The middle graph of a connected graph G, denoted by $M(G)$, is the graph with vertex set $V(M(G)) = V(G) \cup E(G)$ and $uv \in E(M(G))$ if either u and v are adjacent edges of G or one is a vertex of G and the other is an edge of G incident with it.

Theorem 11. *For the graph $G = M(C_p)$, where C_p is a cycle of length p, $\gamma_p(G) = 2p - 4$.*

Proof. For the graph $G = M(C_p)$, $|V(G)| = 2p$ and $\Delta(G) = 4$. By inequality (3), $\gamma_p(G) \leq 2p - 4$.
Let D be a $\gamma_p(G)$-set, which implies $diam(\langle V - D \rangle) \leq 2$. The maximum number of vertices possible in an induced subgraph of G with diameter at most 2 is 5. Thus $|V - D| \leq 5$.
In case $|V - D| = 5$, $\langle V - D \rangle$ is isomorphic to a friendship graph with 5 vertices. But, then, D ceases to be a psd-set of G. Hence $|V - D| \leq 4$, and the result follows. □

4. Another Bound for $\gamma_p(G)$ in terms of Girth of G

In this section we establish another sharp lower bound for point-set domination number of a graph G in terms of its girth $g(G)$.

Theorem 12. *For any graph $G \ncong C_5$ with girth $g(G) \geq 3$, $\gamma_p(G) \geq g(G) - 2$.*

Proof. Let $g(G) = k$ and C be any cycle of length k. We claim that $\gamma_p(G) \geq k-2$. Let D be any psd-set of G. If $g(G) = 3$, then inequality holds trivially. Note that when $\gamma_p(G) = 1$, then there is a vertex $u \in G$ with $deg(u) = |V(G)| - 1$. In that case either G is isomorphic to a star or $g(G) = 3$. Therefore if $g(G) > 3$, we have $\gamma_p(G) \geq 2$. Thus the result holds for $g(G) = 4$. Next consider that girth of G is 5. We shall prove that $\gamma_p(G) \geq 3$. If $|(V-D) \cap V(C)| \leq 2$, then we are through. Therefore let $|(V-D) \cap V(C)| > 2$ and let $C = (x_0, x_1, x_2, x_3, x_4, x_0)$. Since $g(G) = 5$, for any $i \in \mathbb{Z}_5$, $N(x_{i-1}, x_{i+1}) = \{x_i\}$. Hence, for any $i \in \mathbb{Z}_5$, $\{x_{i-1}, x_i, x_{i+1}\} \nsubseteq V - D$. Hence at least two non adjacent vertices of C belong to D, say x_2 and x_4. Further since $G \ncong C_5$ and is a connected graph, there exists $u \in V(G) \setminus V(C)$ such that u is adjacent to at least one vertex of C. If $u \in D$, then $|D| \geq 3$ and we are done. Let $u \notin D$. As $g(G) = 5$, u is adjacent to exactly one vertex of C and in every possible case there exists a vertex in $D \setminus V(C)$ adjacent to u and one vertex of $\{x_0, x_1, x_2\}$. Then in that case $|D| \geq 3$. Hence $\gamma_p(G) \geq 3$ whenever $g(G) = 5$.
Now let $g(G) > 5$ and $C = (x_0, x_1, x_2, \ldots, x_{k-1}, x_0)$. Then since $N(x_{i-1}, x_{i+1}) = \{x_i\}$ for any $i \in \mathbb{Z}_k$, therefore $\{x_{i-1}, x_i, x_{i+1}\} \nsubseteq V - D$. Further, since the distance between any two vertices of $V - D$ is at most 2 (Proposition 1), we have $|(V-D) \cap V(C)| \leq 2$. Hence $\gamma_p(G) \geq k-2$ and the result follows. □

Remark 1. Note that if $G \cong C_5$, then $\gamma_p(G) = 2 = g(G) - 3 < g(G) - 2$. Also the bound for $\gamma_p(G)$ obtained in Theorem 12 is sharp. In fact, for any cycle C_n ($n \neq 5$), $\gamma_p(C_n) = n - 2 = g(C_n) - 2$.

5. Concluding Remarks

In this paper, we focused on lower and upper bounds of point-set domination number of a graph. We succeeded in finding extremal graphs for the lower bound of point-set domination number in terms of its diameter. Although characterizing graphs attaining the upper bound is an interesting but complex problem, we could establish certain classes of graphs such that every graph in that class attains the upper bound. We proved that psd-number of middle graph of a cycle attains the upper bound. It is our hunch that middle graph of a tree and of a unicyclic graph are also extremal graphs for the upper bound of their psd-number.

Conjecture 1. If G is a tree, then $\gamma_p(M(G)) = |V(M(G))| - \Delta(M(G))$.

Conjecture 2. If G is a unicyclic graph, then $\gamma_p(M(G)) = |V(M(G))| - \Delta(M(G))$.

We further raise the problem:

Problem 1. Characterize graphs G such that $\gamma_p(G) = |V(G)| - \Delta(G)$.

Conflict of interest. The authors declare that they have no conflict of interest.

Data Availability. Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

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