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*Research Article*

# Global Malmquist productivity index for evaluation of multi-stage series systems with undesirable and non-discretionary data

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**Abstract:** Data Envelopment Analysis measures relative efficiency, in which the performances of the DMUs in a group are compared. In this approach, an efficient unit in one group may be considered inefficient compared to the units of other groups and vice versa. To solve this weakness, two known productivity indexes, the Malmquist and Luenberger, have been introduced to evaluate units (or systems) from one period to another. The existence of special types of data such as undesirable and non-discretionary in some multi-stage series systems is unavoidable. The evaluation of such systems in the simultaneous presence of the mentioned data and different periods has not been done so far. Therefore, in this study, we have presented a model with a new approach to evaluate them. At the end of the study, we checked the proposed model's ability by providing comparative and structural examples. We have shown that without undesirable and non-discretionary data, the proposed is better than other models. Also, this model has been used for the first time and obtained acceptable results in the presence of these data.

**Keywords:** network data envelopment analysis, malmquist productivity index, evaluation, non-discretionary data, undesirable data.

**AMS Subject classification:** 90C08, 90C90

## 1. Introduction

The performance of any organization is often evaluated as its efficiency in the use of resources. In other words, efficiency indicates how much an organization uses its resources to increase optimal production. Evaluation using efficiency measurement to remain competitive is a continuous performance improvement tool. Charnes et al. (1978) introduced a fractional programming technique to calculate the relative

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efficiency of a unit [12]. Since in the dual form of their model, the observations are enveloped by the production frontier, this model became known as data envelopment analysis ($DEA$). Their model assumed constant returns to scale, and the model with variable return to scale was developed by Banker et al. in 1984. These models are known as $CCR$ and $BCC$ respectively [7]. After that, many studies on methodologies and applications of $DEA$ have been done and the number of studies has been growing. Decision-making units may have different internal divisions, in which different processes are carried out. Traditional DEA methods ignore these processes and consider the system as a black box that prevents access to valuable information and yields incorrect efficiency scores. Thus, to study the performance of a $DMU$, it is necessary to study its constituents, so that the cause of any inefficiencies can be identified, and the measured efficiency will be meaningful. This idea was discussed by Charnes et al. in 1986, where they found that army recruitment had two stages, creating awareness through advertising and signing contracts [11].

Breaking large operations into smaller parts makes the efficiency score more realistic. It also helps us identify the real effects of factors. For this reason, Fare and Grosscopf proposed the network data envelopment analysis ($NDEA$), considering the operation of component processes in calculating the efficiency of the system. Cook et al. introduced a multi-stage model in 2010, in which each stage could have an exogenous input and a final output, known as general network systems [14]. Very soon the models were developed to measure the efficiency of network production systems, and the applications have been made in the real world. Kao in 2014 presented details of many related models and applications in $NDEA$ and gave a good overview of this topic [25]. Tayyebi and Amirteimoori proposed a $DEA$ approach on extension shortest path problem [46].

In many real-world issues that are examined using DEA, data are not always normal. Sometimes systems have special data that we have to deal with in their ways. Some special data without physical value, such as fuzzy, stochastic, interval, ordinal data, etc., can be present in the production process. Researchers have conducted many studies on these data using the DEA approach. Fallah et al. studied discriminant analysis and data envelopment analysis using specific data (2020) [15]. Pourmahmoud and Norouzi Bene (2022) provided a new model for evaluating and ranking DMUs with ordinal data. The general idea of their model is assigning real values to ordinal data with the new approach. Furthermore, in their study, another new model for ranking efficient units is presented with the main idea of changes in controlled efficiency [38].

Also, some multi-stage series systems unavoidably include special types of data, such as undesirable and non-discretionary. The studies have been done to evaluate systems in the separate presence of mentioned data by researchers. Omrani et al. (2022) developed an NDEA model with negative inputs and undesirable outputs [33]. Shirvani and Azizi presented a model by developing a two-stage network data envelopment analysis model with desirable and undesirable outputs (2022). The contribution of their research is to develop a model for evaluating the efficiency of a two-stage production system with both desirable and undesirable output using the network model

of Khalili and Shahmiri [43]. Li et al. (2022) proposed a new DEA approach for measuring the eco-inefficiency of two-stage structures with undesirable intermediate measures and then applied the approach to 30 Chinese provincial regions with production and pollution treatment activities [30].
Taleb et al. (2019) proposed a two-stage approach of super efficiency slack-based measure in non-discretionary factors and mixed integer requirements. The practicability of their proposed approach was tested using empirical data of Malaysian community colleges [44]. Galagedera (2019) formulated network data envelopment analysis (DEA) output-oriented models to compute overall and stage-level performance. In his study, a DEA model to determine the frontier projection of inefficient MFs is also developed [19].
In 2022, Pourmahmoud and Norouzi tried to extend of CCR model to evaluate two-stage network systems in the presence of undesirable and non-discretionary data. They evaluated general two-stage systems by defining a parameter for each division of the system. It should be noted that this model was only for evaluating systems in a specific period [37]. But, in this study, we evaluate general multi-stage series network systems with a new approach and definition of the global frontier in the presence of undesirable and non-discretionary factors. The models presented in this study and the previous ones are different.
So far, researchers have not evaluated multi-stage series systems in the simultaneous presence of undesirable and non-discretionary data at different periods. Thus, the main goal of this study is to evaluate such systems in the presence of mentioned data at different periods. It aims to recognize the impact of divisions on productivity changes in a system. The proposed model can determine if a system or division improved or regressed from one period to another. The proposed model assigns a separate contraction or expansion parameter to each input or output in each period. The assignment depends on the model's type in terms of input or output orientation. In addition, the proposed model can also detect how the divisions affect the system's productivity changes. We calculated the productivity changes of the systems and divisions during period T. We used the input-oriented global Malmquist productivity index.
In the following, the second section presents the basic conceptions. Section 3 provides the proposed model. In the fourth section, we examine and analyze the results. We do this by presenting comparative and numerical examples. Finally, we present the conclusion in section 5.

## 2. Basic conceptions

### 2.1. Malmquist productivity index (MPI)

The Malmquist productivity index was introduced by Chavez et al. (2016) based on the idea proposed by Professor Sten Malmquist as a quality index for analyzing the consumption of production resources. In addition to a productivity index, input and output indices were also defined [4]. They considered the problem of comparing two DMUs k and l when k and l may represent the same DMU at two different times or different DMUs at either the same time or different times, and defined three follow indices.

### 2.1.1. Input, output, and productivity index

Let $T^k$ be the production technology of $DMU_k$ which produces the outputs $Y^k$ from the inputs $X^k$ ; then $T^k$ will be as follows [26]:

$$T^k = \{(X_k, Y_k)|Y_k \geq 0, can\, be\, produced\, by X_k \geq 0\} \tag{2.1}$$

The input distance function based on the technology of $DMU_k$ is:

$$D_I^k(Y, X) = Max\{\delta|(\frac{X}{\delta}, Y) \in T^k\} \tag{2.2}$$

Or:

$$D_I^k(Y, X) = [Min\{\delta|(\delta X, Y) \in T^k\}]^{-1} \tag{2.3}$$

Chavez et al. defined the $DMU_k$ Malmquist input index as:

$$M_I^k(X^k, X^l) = \frac{D_I^k(Y^k, X^k)}{D_I^k(Y^k, X^l)} \tag{2.4}$$

Since $D_I^k(Y^k, X^k) = 1$ then $M_I^k(X^k, X^l) = \frac{1}{D_I^k(Y^k, X^l)} = Min\{\delta|(\delta X, Y) \in T^k\}$ , which is the minimum factor $\delta$ required to expand the input vector of $DMU_l$ onto the production surface of $DMU_k$ , the output vector is that of $DMU_k$ .
It is reminded that if $M_I^k(X^k, X^l) > 1$ then the input vector of $DMU_k$ is larger than that of $DMU_l$ , from the perspective of $DMU_k$ 's technology.
The output index can be discussed in the same way as the input index.
In addition to input and output indexes, Chavez et al. [26] also defined productivity indexes based on input and output to measure productivity differences between two DMUs. They defined Malmquist input-based productivity index of $DMU_k$ as:

$$M_I^k(X^k, Y^k, X^l, Y^l) = \frac{D_I^k(Y^l, X^l)}{D_I^k(Y^l, X^l)} \tag{2.5}$$

Since $D_I^k(Y^k, X^k) = 1$ then:

$$M_I^k(X^k, Y^k, X^l, Y^l) = D_I^k(Y^l, X^l) = [Min\{\delta|(\delta X^l, Y^l) \in T^k\}]^{-1} \tag{2.6}$$

$M_I^k(X^k, Y^k, X^l, Y^l)$ is the maximum input contraction factor ($\delta^*$), such that the contracted input for $DMU_l$ and the output vector lie on the production surface of $DMU_k$. if $M_I^k(X^k, Y^k, X^l, Y^l) > 1$ then $DMU_k$ has a higher productivity level than

$DMU_l$. Chavez et al. similarly defined Malmquist output-based productivity index as:

$$M_o^k(X^k, Y^k, X^l, Y^l) = \frac{D_o^k(Y^k, X^k)}{D_o^k(Y^l, X^l)} \tag{2.7}$$

Since $D_o^k(Y^k, X^k) = 1$ then:

$$M_o^k(X^k, Y^k, X^l, Y^l) = \frac{1}{D_o^k(Y^l, X^l)} = [Max\{\delta|(X^l, \delta Y^l) \in T^k\}]^{-1} \tag{2.8}$$

$M_o^k(X^k, Y^k, X^l, Y^l)$ is the maximum output expention factor ($\delta^*$), such that the expention output for $DMU_l$ and the input vector lie on the production surface of $DMU_k$. if $M_o^k(X^k, Y^k, X^l, Y^l) > 1$ then $DMU_k$ has a higher productivity level than $DMU_l$.

#### 2.1.2. MPI

The MPI is used to evaluate efficiency in different periods. It identifies the regress and progress of the DMUs during periods. Fare et al. (1994) analyzed the $MPI$ based on technology and efficiency variations [17]. Yao et al. (2016) introduced the cost-based MPI. They used the meta-frontier non-radial Malmquist CO2 emission performance index (MNMCPI) to estimate the changes in China's CO2 emission performance [52]. Chen and Golley (2014) used a Directional Distance Function (DDF) and the Malmquist-Luenberger Productivity Index to estimate the changing patterns of 'green' total factor productivity (GTFP) growth of 38 Chinese industrial sectors during 1980–2010 [13]. The Malmquist output-based productivity index defined by Fare et al. does not need a specific function for the technology. This approach can also be defined from the input side. The Malmquist output-based productivity index based on the technology of period t is:

$$MPI_o^t(X^t, Y^t, X^{t+1}, Y^{t+1}) = \frac{D_o^t(Y^{t+1}, X^{t+1})}{D_o^t(Y^t, X^t)} \tag{2.9}$$

$D_o^t(Y^t, X^t)$ represents the efficiency of DMU at period based on technology of period t and $D_o^t(Y^{t+1}, X^{t+1})$ denotes the efficiency of DMU within the period based on technology of period t+1. $[D_o^t(Y^{t+1}, X^{t+1})]^{-1}$ can be calculated from following linear program [26]:

$$\begin{aligned}
[D_o^t(Y^{t+1}, X^{t+1})]^{-1} = {} & Max\ \phi \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j X_{ij}^t \le X_{ij}^{t+1}, i = 1, \cdots, m \\
& \sum_{j=1}^{n} \lambda_j Y_{rj}^t \ge \phi Y_{rj}^{t+1}, r = 1, \cdots, s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \ge 0, i = 1, \cdots, n \\
& \phi \text{ unrestricted in sign}
\end{aligned} \tag{2.10}$$

The value of $\phi$ can be greater than, equal to, or less than one. In this case, if $MPI_o^t > 1$, the efficiency of this $DMU$ has increased from period $t$ to period $t+1$. If $MPI_o^t < 1$then the efficiency of this DMU has decreased from period $t$ to period $t+1$. The efficiency remains the same if it is equal to one. The Malmquist output-based productivity index based on the technology of period $t+1$ is [26]:

$$MPI_o^{t+1}(X^t, Y^t, X^{t+1}, Y^{t+1}) = \frac{D_o^{t+1}(Y^{t+1}, X^{t+1})}{D_o^{t+1}(Y^t, X^t)} \tag{2.11}$$

Where $D_o^{t+1}(Y^t, X^t)$ can be calculated from following linear program [26]:

$$\begin{aligned}
[D_o^{t+1}(Y^t, X^t)]^{-1} = & \; Max\ \phi \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j X_{ij}^{t+1} \le X_{ij}^t, i = 1, \cdots, m \\
& \sum_{j=1}^{n} \lambda_j Y_{rj}^{t+1} \ge \phi Y_{rj}^t, r = 1, \cdots, s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \ge 0, i = 1, ..., n \\
& \phi \text{ unrestricted in sign}
\end{aligned} \tag{2.12}$$

The values of $MPI_o^t$ and $MPI_o^{t+1}$ may not be the same but they usually have the same trend of being greater or less than one. But, different technologies can yield different results in certain cases. Fare, Grosskopf, Norris, and Zhang (1994) suggested taking the geometric mean of $MPI_o^t$ and $MPI_o^{t+1}$ as the final MPI to avoid different results [18],

$$MPI_o^{FGNZ}(X^t, Y^t, X^{t+1}, Y^{t+1}) = [\frac{D_o^t(Y^{t+1}, X^{t+1})}{D_o^t(Y^t, X^t)} \times \frac{D_o^{t+1}(Y^{t+1}, X^{t+1})}{D_o^{t+1}(Y^t, X^t)}]^{\frac{1}{2}} \tag{2.13}$$

This index calculates changes in productivity. It can be decomposed into the following useful terms:

$$MPI_o^{FGNZ}(X^t, Y^t, X^{t+1}, Y^{t+1}) = [\frac{D_o^t(Y^{t+1}, X^{t+1})}{D_o^t(Y^t, X^t)} \times \frac{D_o^{t+1}(Y^{t+1}, X^{t+1})}{D_o^{t+1}(Y^t, X^t)}]^{\frac{1}{2}} = (\text{EC}) \times (\text{TC}) \tag{2.14}$$

The $MPI_o^{FGNZ}$ is not circular and the distance measures can be infeasible under variable returns to scale. For these reasons, Pastor and Lovell introduced a global

$MPI$ in 2005 [34] and 2007 [35]. They used the observations of all periods to construct a frontier. The global Malmquist productivity index is circular. It provides unique measures of productivity change. All observations were used to construct the frontier. Thus, this model is always feasible. The global $MPI$ for a $DMU$ between periods $t$ and $t+a$ is defined as [26]:

$$MPI_o^G(X^t, Y^t, X^{t+a}, Y^{t+a}) = \frac{D_o^G(Y^{t+a}, X^{t+a})}{D_o^G(Y^t, X^t)} \tag{2.15}$$

Where

$$\begin{aligned}
[D_o^G(Y^k, X^k)]^{-1} = &\ Max\ \phi \\
\text{s.t.} \quad & \sum_{t=1}^{T}\sum_{j=1}^{n} \lambda_j^t X_{ij}^{t+1} \leq X_{io}^k, i = 1, \cdots, m \\
& \sum_{t=1}^{T}\sum_{j=1}^{n} \lambda_j{}^t Y_{rj}^{t+1} \geq \phi Y_{ro}^k, r = 1, \cdots, s \\
& \sum_{t=1}^{T}\sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j^t \geq 0, i = 1, \cdots, n, t = 1, \cdots, T \\
& \phi \text{ unrestricted in sign}
\end{aligned} \tag{2.16}$$

If $MPI_o^G(X^t, Y^t, X^{t+a}, Y^{t+a}) > 1$ then the productivity of this $DMU$ has increased from period $t$ to period $t+1$ .

In the following, we will provide explanations about undesirable and non-discretionary data. We will try to evaluate multi-stage series network systems by developing model (2.16) in the presence of undesirable and non-discretionary data.

### 2.2. Undesirable and non-discretionary data

In real-world issues, data is not always desirable or discretionary. Sometimes undesirable and non-discretionary, or both, are present in the system. Traditional $DEA$ methods improve unit efficiency by reducing inputs or expanding outputs. But, reduced input and expanded output also include desirable and and non-discretionary data. Thus, these methods ignore them and incorrect results may occur during calculations. Several approaches have been introduced for dealing with these factors. These include data transformation, input-output exchange, slacks-based measures, weak disposability for undesirable data, and fixing non-discretionary data [26].

#### 2.2.1. Undesirable data

Undesirable outputs were proposed by Pittman in 1983 [36]. After that many researchers studied this type of outputs. Sifford and Zhou (2002) presented a model with desirable and undesirable data based on BCC model, in which the undesirable outputs were multiplied by a negative [41]. The challenge of this model was investigated by Fare and Grosskopf which was obtaining different answers, and was further accepted by Sifford and Zhou. They solved this problem by defining a directed distance function in 2004 [16]. Jahanshahloo et al. (2004) presented multi-objective linear programming to solve problems with undesirable data. They investigated inputs/outputs estimate in the presence of undesirable factors [23]. KordRostami and Amirteimoori (2005) presented a multi-stage model in which undesirable variables with a negative sign were used in the calculation of weights [28]. Amirteimoori et al. (2006) used a model with the aim of improving efficiency by increasing undesirable inputs and reducing undesirable outputs [6]. Akhtar et al. (2013) presented a model to minimize undesirable and maximize desirable outputs [3]. Homayounfar and Amirteimoori (2016) applied a fuzzy network method based on DEA in the presence of desirable and undesirable outputs in their study [20]. Wu et al. (2016) provided an approach for analyzing the reuse of undesirable intermediate outputs in a two-stage production process with a shared resource [51]. Madadi et al. (2018) expanded a resource allocation model for evaluating 25 branches of an Iranian Tejarat bank in the presence of undesirable data [31]. Seihani Parashkouh et al. (2020) proposed two non-linear technologies based on weak disposability definitions for two-stage systems with undesirable data [42]. Yu et al. (2020) presented an improved matrix-type network data envelopment analysis (NDEA) model with undesirable output to evaluate the eco-efficiency of China's 30 provinces [53].

#### 2.2.2. Non-discretionary data

One of the advantages of the DEA approach is identifying targets for inefficient DMUs to become efficient. This is based on the reduction of the inputs and expansion of the outputs. This approach is not useful when some inputs or outputs are non-discretionary. It yields incorrect efficiency scores.

The first study on non-discretionary data was performed by Banker and Murray in 1986 [8]. Their model evaluated units by comparing them in more stringent environments in terms of non-discretionary factors. Their other model (1986), which was based on the idea of discretionary or non-discretionary condition of data, is currently one of the most widely used models in this field [9]. In 1991, using regression analysis, Ray investigated the effect of non-discretionary factors as independent variables on unit efficiency [39]. In 1997, Ruggiero presented a model that selects the reference set from units with a stringent environment or at least a similar environment in the presence of non-discretionary data [40]. Hosseinzadeh et al. (2007) employed the super efficiency approach in DEA in the presence of non-discretionary inputs [21]. Jahanshahloo et al. used a non-radial DEA to discuss non-discretionary data in 2007. Their research proposed a new ranking system for extreme efficient DMUs based upon the omission of these efficient DMUs from reference set of the inefficient DMUs [22]. Camanho et al. (2009) presented a model that treated non-discretionary data depending on their classification as internal or external. They proposed an enhanced DEA model that accommodates non-discretionary inputs and outputs and treats them differently depending on their classification as internal or external to the production process [10]. Abri and Fallah Jelodar (2012) proposed a linear model by considering non-discretionary factors and reviewed previous models [1].

### 2.3. General multi-stage systems

Many systems have a multi-stage structure. Basic material goes through several stations to become the final outputs. In this system, a stage may have several divisions connected in different structures. The system reviewed in the conventional network $DEA$ is composed of some divisions connected in series. Only one division is in each stage. The simplest type of these systems is the basic series structure. The next division consumes the outputs of each middle division as input. In this type, the first division has an exogenous input, and only the last division has a final output. These systems are known as basic multi-stage systems. Figure 1 shows the structure of the basic multi-stage system [26]. In general multi-stage series systems, some interme-

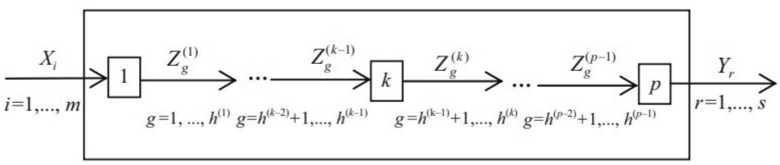


**Figure 1.** **Structure of the basic series system**

diate products may exit the system. Any division may need input from outside to become the final product [26]. The structure of these systems will be reviewed in this study, displayed in Figure 2. We will discuss the efficiency of this structure in the simultaneous presence of undesirable and non-discretionary data. We will also evaluate the productivity change of a system from one period to another. Researchers

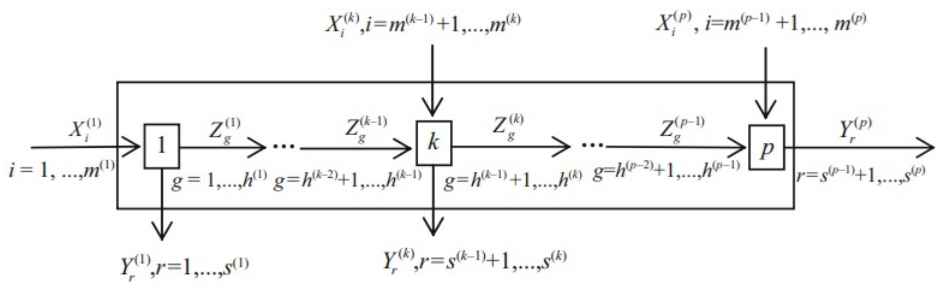


**Figure 2.** **Structure of the general series system**

have performed several studies on NDEA. These studies aimed to calculate the efficiency of multi-stage systems with a series structure. Troutt et al. (2001) proposed a value-based model of maximizing the throughput per unit of input at the first process for the basic multi-stage system [48]. Tone and Sahoo (2003) modified the system distance measure model. They used it to investigate returns to scale in the presence of indivisibilities in all multi-stage production processes [47]. Amirteimoori and Kordrostami (2005) [5] as well as Kordrostami and Amirteimoori (2005) [6] proposed models to measure the performance of a series system. Tsutsui and Goto (2009) used SBM model to evaluate 90 vertically integrated electric power companies in the US [49]. Wei et al. (2011) discussed a basic multi-stage system without intermediate products [50]. Kao and Liu (2011) measured the performance of 22 commercial banks in Taiwan for the period 2009–2011 [27]. Lee and Johnson (2012) applied the relational model of Kao (2009a) [29] for general network systems, with a modification for variable returns to scale, to examine the performance of 15 US Airlines. Matthews (2013) studied the risk management and managerial efficiencies of 15 banks in China with an SBM model [32]. Kao (2014) attached a dummy process to each original one to carry the exogenous inputs and outputs to alter the general series structure to a basic multi-stage one [24]. Ahmad Khanlou Gharakhanlou et al. (2023) proposed an approach to calculate cost, revenue and profit Efficiency in multi-period network [2]. These studies on $NDEA$ are considerable. However, researchers have not evaluated such systems in the presence of both undesirable and non-discretionary data. In this study, we evaluated these systems using global MPI in different periods. We defined different distance parameters for any input. We did this for any division at each period. This identifies the regress and progress of the DMUs during periods.

## 3. Proposed model

In this section, we present a model for evaluating multi-stage network systems with series structure in the simultaneous presence of undesired and non-discretionary data. We use the input-output exchange approach for undesirable factors and keep the non-discretionary factors constant to manage them. We assume multi-stage series systems with $k$ divisions for evaluation. In this case, we present the proposed model assuming the following assumptions:
Suppose for the division $p$ of system $j, j = 1, ..., n$
Number of inputs= $m_p, p = 1, ..., k$.
Number of desirable inputs=$d_p, p = 1, ..., k$.
Number of undesirable inputs = $q_p, p = 1, ..., k$.
Number of non-discretionary inputs= $t_p, p = 1, ..., k$.
Where $d_p + q_p + t_p = m_p, p = 1, ..., k$.
Number of outputs= $s_p, p = 1, ..., k$.
Number of desirable outputs=$l_p, p = 1, ..., k$.
Number of undesirable outputs =$u_p, p = 1, ..., k$.
Number of non-discretionary outputs=$v_p, p = 1, ..., k$.
Where $l_p + u_p + v_p = s_p, p = 1, ..., k$.
Number of intermediate products = $h_p, p = 1, ..., k$.
The idea of the global $MPI$ is to use all observations at all of the periods to construct the frontier. The input distance model for a system with $CRS$ technology at period

$a \in \{1, 2, \ldots, T\}$ suggested as follows:

$$
\begin{aligned}
[D_I^G(O^a)] =& Min\left(\sum_{p=1}^{k} w_p \theta_p^a\right) \\
\text{s.t.} \quad & \sum_{t=1}^{T}\sum_{j=1}^{n} (\lambda_j^p)^t (x_{ij}^p)^t \le \theta_p^a (x_{io}^p)^a, i = 1, \cdots, d_p, p = 1, \cdots, k \\
& \sum_{t=1}^{T}\sum_{j=1}^{n} (\lambda_j^p)^t (x_{ij}^p)^t \ge (x_{io}^p)^a, i = d_p + 1, \cdots, d_p + q_p, p = 1, \cdots, k \\
& \sum_{t=1}^{T}\sum_{j=1}^{n} (\lambda_j^p)^t (x_{ij}^p)^t \le (x_{io}^p)^a, i = d_p + q_p + 1, \cdots, m_p, p = 1, \cdots, k \\
& \sum_{t=1}^{T}\sum_{j=1}^{n} (\lambda_j^p)^t (y_{rj}^p)^t \ge (y_{ro}^p)^a, r = 1, \cdots, l_p, p = 1, \cdots, k \qquad (3.1) \\
& \sum_{t=1}^{T}\sum_{j=1}^{n} (\lambda_j^p)^t (y_{rj}^p)^t \le \theta_p^a (y_{ro}^p)^a, r = l_p + 1, \cdots, l_p + u_p, p = 1, \cdots, k \\
& \sum_{t=1}^{T}\sum_{j=1}^{n} (\lambda_j^p)^t (y_{rj}^p)^t \ge (y_{ro}^p)^a, r = l_p + u_p + 1, \cdots, s_p, p = 1, \cdots, k \\
& \sum_{j=1}^{n} (\lambda_j^p)^t (z_{g_j}^p)^t \ge \sum_{j=1}^{n} (\lambda_j^{p+1})^t (z_{g_o}^{p+1})^a, g_j^p = h_j^{p-1} + 1, ..., h_j^p, \\
& p = 1, \cdots, k-2, t = 1, 2, \cdots, T, h_j^0 = 0 \\
& \sum_{j=1}^{n} (\lambda_j^{p+1})^t (z_{g_j}^{p+1})^t \le (z_{g_o}^p)^a, g_j^p = h_j^{p-1} + 1, \cdots, h_j^p, p = k-1, \\
& (\lambda_j^p)^t \ge 0, i = 1, \cdots, n, t = 1, \cdots, T
\end{aligned}
$$

Under $VRS$ technology, the constraint $\sum_{j=1}^{n} (\lambda_j^p)^t = 1$ , should be added. $w_p, p = 1, ..., k$ are the weights assigned to each division by the system manager where , $\sum_{p=1}^{k} w_p = 1, w_p \ge 0, p = 1, ..., k$. The input distance parameter $(\theta_p^a)$ is the minimum contraction factor that can keep the inputs of the division p of system $o$ being evaluated in the production possibility set.

The first 3 constraints are related to the inputs and the second 3 constraints are linked to the outputs of division p. The first constraint corresponds to the desirable inputs, the second to the undesirable inputs, and the third to the non-discretionary inputs. The constraints of outputs are also presented similarly. The seventh and eighth restrictions are related to the intermediate products. This model ensures that the intermediate product as an output is greater than or equal to that as an input. The global $MPI$ for system $o$ between periods $t$ and $t+a$ is defined as:

$$MPI_I^G(o^t, o^{t+a}) = \frac{D_I^G(o^{t+a})}{D_I^G(o^t)} \tag{3.2}$$

If the value of $MPI_I^G(o^t, o^{t+a})$ is greater than 1, the productivity of this system has increased from period $t$ to $t+a$. The global MPI for division $p$ of the system $o$ between periods $t$ and $t+a$ is defined as:

$$MPI_I^G(p_o^t, p_o^{t+a}) = \frac{D_I^G(p_o^{t+a})}{D_I^G(p_o^t)} = \frac{\theta_p^{t+a}}{\theta_p^t} \tag{3.3}$$

As shown in the following, the value of $MPI_I^G$ for the period $(t, t+a+b)$ is equal to the multiplication of it for the periods $(t, t+a)$ by $(t+a, t+a+b)$. Therefore, in this case the circularity property is satisfied.

$$\begin{aligned}
MPI_I^G(o^t, o^{t+a}) &= \frac{D_I^G(o^{t+a})}{D_I^G(o^t)} \\
MPI_I^G(o^{t+a}, o^{t+a+b}) &= \frac{D_I^G(o^{t+a+b})}{D_I^G(o^{t+a})} \\
MPI_I^G(o^t, o^{t+a}) \times MPI_I^G(o^{t+a}, o^{t+a+b}) &= \frac{D_I^G(o^{t+a})}{D_I^G(o^t)} \times \frac{D_I^G(o^{t+a+b})}{D_I^G(o^{t+a})} \\
&= \frac{D_I^G(o^{t+a+b})}{D_I^G(o^t)} \\
&= MPI_I^G(o^t, o^{t+a})
\end{aligned} \tag{3.4}$$

**Theorem 1.** *Model (3.1) is feasible and bounded.*

*Proof.* To prove the feasibility of model (3.1), we just need to find a feasible solution for this model. Assuming that the system $o$ at period $z$ is under evaluation, we consider the following answer:

$$\begin{aligned}
&(\lambda_j^p)^t = 0, t = 1, 2, \cdots, T, p = 1, \cdots, k, j \neq o \\
&(\lambda_o^p)^t = 0, p = 1, \cdots, k, t \neq z \\
&(\lambda_o^p)^z = 0, p = 1, \cdots, k \\
&\theta_p^z = 1, p = 1, \cdots, k
\end{aligned} \tag{3.5}$$

It is clear that (3.5) is an answer for model (3.1) and is satisfied in all constraints. In other words, there is at least one answer for this model. Assuming this answer, the

value of the objective function will be equal to one. Thus, the proposed model is feasible. On the other hand, since the objective function of the model is of minimization type and the value of the objective function is equal to 1 for a possible answer, then the optimal value of objective function will be less than or equal to 1.
In other words, if the optimal value of the model is $\theta^*$ then $\theta^* \leq \theta \leq 1$ is established for all values of objective function $\theta$. Also, the value of the objective function cannot be zero because if:

$$\theta_p^z = 0, p = 1, ..., k. \tag{3.6}$$

In this case, from first set of constraints the following result is obtained as:

$$\sum_{t=1}^{T} \sum_{j=1}^{n} (\lambda_j^p)^t (x_{ij}^p)^t \leq 0 \tag{3.7}$$

With these conditions, the factors $(x_{ij}^p)^t$ must be negative which is a contradiction. So, none of the parameters $\theta_p^z$ can be set to zero. For the same reason, none of the parameters $\theta_p^z$ can take a negative value. Thus, the finiteness of the model is also proved. □

**Definition 1.**

a) The evaluated system$o$ at period $z$ in model (3.1) is efficient when the value of the objective function $(D_I^G(o^z))$ is equal to 1.

b) The division $p$ of the evaluated system at period $z$ in model (3.1) is efficient when $\theta_p^z$ is equal to 1.

We introduced a model in section 3. It can evaluate multi-stage series systems. It does this in the presence of undesired and non-discretionary data. To check the performance of the model on the issues, two examples are provided in the next section. The first example compares the proposed model to other models in the evaluation of systems without undesirable and non-discretionary data. The second is a structural example designed by the authors. It defines 10 hypothetical systems in the presence of undesirable and uncontrollable data.

## 4. Examples

### 4.1. Comparative example

In this section, we present the example that Tawana et al. examined in their study entitled "A Malmquist productivity index for network production systems in the energy sector" [45]. We compare this example with our proposed model and compare the results. Their proposed method has been applied to measure the productivity of several Iranian oil refineries. After identifying the main factors determining the productivity of these refineries, the operation of nine of them is analyzed using data from the 2015–2016 period. The inputs and outputs data for the years 2015 and 2016 are presented in Tables (1) to (4), respectively. The results obtained from their study and our proposed model are presented in two main parts. First, the total efficiency scores, including those of the first and second stages, are provided. Furthermore, the $MPI$ is calculated for the entire process and each one of its stages. Note that in this example, the values of the weights are considered as $w_1 = w_2 = \frac{1}{2}$.

**Table 1.** **Input-output data in 2015 (stage 1)**

| Criteria | Abadan | Bandarabas | Arak | Esfehan | Tehran | Tabriz | Shiraz | Lavan | Kermanshah |
|---|---|---|---|---|---|---|---|---|---|
| $x_1^1$ | 0.2210 | 0.1707 | 0.1403 | 0.2071 | 0.1353 | 0.0623 | 0.0300 | 0.0212 | 0.0120 |
| $x_2^1$ | 0.1508 | 0.1423 | 0.1774 | 0.2027 | 0.1875 | 0.0625 | 0.0441 | 0.0189 | 0.0139 |
| $x_3^1$ | 0.2258 | 0.1853 | 0.1448 | 0.1621 | 0.1419 | 0.0637 | 0.0347 | 0.0290 | 0.0127 |
| $x_4^1$ | 0.0783 | 0.0756 | 0.2040 | 0.1529 | 0.1234 | 0.1080 | 0.0736 | 0.1277 | 0.0566 |
| $x_5^1$ | 0.2918 | 0.1257 | 0.1242 | 0.1259 | 0.1088 | 0.0686 | 0.0643 | 0.0460 | 0.0445 |
| $z_1^1$ | 0.2946 | 0.1123 | 0.0734 | 0.1668 | 0.1321 | 0.0505 | 0.0104 | 0.1559 | 0.0039 |
| $z_2^1$ | 0.2551 | 0.2034 | 0.1274 | 0.1598 | 0.1125 | 0.0471 | 0.0478 | 0.0293 | 0.0177 |
| $z_3^1$ | 0.1619 | 0.2272 | 0.1827 | 0.1845 | 0.1129 | 0.0540 | 0.0431 | 0.0195 | 0.0142 |
| $z_4^1$ | 0.3111 | 0.1197 | 0.1347 | 0.1682 | 0.1531 | 0.0326 | 0.0412 | 0.0282 | 0.0112 |
| $z_5^1$ | 0.2440 | 0.1883 | 0.1346 | 0.1588 | 0.1313 | 0.0666 | 0.0108 | 0.0340 | 0.0177 |
| $z_6^1$ | 0.2098 | 0.1896 | 0.1588 | 0.1687 | 0.1393 | 0.0759 | 0.0320 | 0.0157 | 0.0103 |
| $z_7^1$ | 0.1275 | 0.1372 | 0.2782 | 0.1765 | 0.1712 | 0.0834 | 0.0260 | 0.0001 | 0.0001 |
| $y_1^1$ | 0.0653 | 0.0002 | 0.0002 | 0.3922 | 0.4818 | 0.0279 | 0.0002 | 0.0002 | 0.0002 |
| $y_2^1$ | 0.0001 | 0.1151 | 0.1316 | 0.3947 | 0.1732 | 0.0830 | 0.1022 | 0.0001 | 0.0001 |

**Table 2.** **Input-output data in 2015 (stage 2)**

| *Criteria* | *Abadan* | *Bandarabas* | *Arak* | *Esfehan* | *Tehran* | *Tabriz* | *Shiraz* | *Lavan* | *Kermanshah* |
|---|---|---|---|---|---|---|---|---|---|
| $x_1^2$ | 0.0016 | 0.0016 | 0.0016 | 0.5752 | 0.0441 | 0.0964 | 0.1748 | 0.0016 | 0.1029 |
| $x_2^2$ | 0.3929 | 0.0008 | 0.0032 | 0.2838 | 0.0089 | 0.0946 | 0.1447 | 0.0008 | 0.0703 |
| $z_1^2$ | 0.2946 | 0.1123 | 0.0734 | 0.1668 | 0.1321 | 0.0505 | 0.0104 | 0.1559 | 0.0039 |
| $z_2^2$ | 0.2551 | 0.2034 | 0.1274 | 0.1598 | 0.1125 | 0.0471 | 0.0478 | 0.0293 | 0.0177 |
| $z_3^2$ | 0.1619 | 0.2272 | 0.1827 | 0.1845 | 0.1129 | 0.0540 | 0.0431 | 0.0195 | 0.0142 |
| $z_4^2$ | 0.3111 | 0.1197 | 0.1347 | 0.1682 | 0.1531 | 0.0326 | 0.0412 | 0.0282 | 0.0112 |
| $z_5^2$ | 0.2440 | 0.1883 | 0.1346 | 0.1588 | 0.1313 | 0.0666 | 0.0108 | 0.0340 | 0.0177 |
| $z_6^2$ | 0.2098 | 0.1896 | 0.1588 | 0.1687 | 0.1393 | 0.0759 | 0.0320 | 0.0157 | 0.0103 |
| $z_7^2$ | 0.1275 | 0.1372 | 0.2782 | 0.1765 | 0.1712 | 0.0834 | 0.0260 | 0.0001 | 0.0001 |
| $y_1^2$ | 0.1965 | 0.1399 | 0.1536 | 0.2338 | 0.1648 | 0.0653 | 0.0292 | 0.0078 | 0.0091 |
| $y_2^2$ | 0.2572 | 0.1742 | 0.1523 | 0.1620 | 0.1187 | 0.0546 | 0.0410 | 0.0237 | 0.0162 |
| $y_3^2$ | 0.1905 | 0.1623 | 0.0501 | 0.1976 | 0.2044 | 0.1213 | 0.0110 | 0.0001 | 0.0629 |
| $y_4^2$ | 0.1949 | 0.1662 | 0.1459 | 0.2134 | 0.1317 | 0.0641 | 0.0363 | 0.0322 | 0.0110 |
| $y_5^2$ | 0.2750 | 0.2134 | 0.1427 | 0.1602 | 0.0945 | 0.0505 | 0.0235 | 0.0243 | 0.0159 |

**Table 3.** **Input-output data in 2016 (stage 1)**

| Criteria | Abadan | Bandarabas | Arak | Esfehan | Tehran | Tabriz | Shiraz | Lavan | Kermanshah |
|---|---|---|---|---|---|---|---|---|---|
| $x_1^1$ | 0.2192 | 0.1728 | 0.1342 | 0.2087 | 0.1374 | 0.0630 | 0.0290 | 0.0242 | 0.0116 |
| $x_2^1$ | 0.1463 | 0.1384 | 0.2347 | 0.2172 | 0.1279 | 0.0667 | 0.0436 | 0.0144 | 0.0108 |
| $x_3^1$ | 0.2258 | 0.1853 | 0.1448 | 0.1621 | 0.1419 | 0.0637 | 0.0347 | 0.0290 | 0.0127 |
| $x_4^1$ | 0.0783 | 0.0756 | 0.2040 | 0.1529 | 0.1234 | 0.1080 | 0.0736 | 0.1277 | 0.0566 |
| $x_5^1$ | 0.3040 | 0.1349 | 0.1218 | 0.1157 | 0.1053 | 0.0708 | 0.0618 | 0.0447 | 0.0407 |
| $z_1^1$ | 0.2946 | 0.1123 | 0.0734 | 0.1668 | 0.1321 | 0.0505 | 0.0104 | 0.1559 | 0.0039 |
| $z_2^1$ | 0.2551 | 0.2034 | 0.1274 | 0.1598 | 0.1125 | 0.0471 | 0.0478 | 0.0293 | 0.0177 |
| $z_3^1$ | 0.1619 | 0.2272 | 0.1827 | 0.1845 | 0.1129 | 0.0540 | 0.0431 | 0.0195 | 0.0142 |
| $z_4^1$ | 0.3111 | 0.1197 | 0.1347 | 0.1682 | 0.1531 | 0.0326 | 0.0412 | 0.0282 | 0.0112 |
| $z_5^1$ | 0.2440 | 0.1883 | 0.1346 | 0.1588 | 0.1313 | 0.0666 | 0.0108 | 0.0340 | 0.0177 |
| $z_6^1$ | 0.2098 | 0.1896 | 0.1588 | 0.1687 | 0.1393 | 0.0759 | 0.0320 | 0.0157 | 0.0103 |
| $z_7^1$ | 0.1275 | 0.1372 | 0.2782 | 0.1765 | 0.1712 | 0.0834 | 0.0260 | 0.0001 | 0.0001 |
| $y_1^1$ | 0.0655 | 0.0345 | 0.0027 | 0.4035 | 0.4736 | 0.0194 | 0.0002 | 0.0002 | 0.0002 |
| $y_2^1$ | 0.0001 | 0.1098 | 0.0593 | 0.4473 | 0.2005 | 0.0849 | 0.0978 | 0.0001 | 0.0001 |

**Table 4.** **Input-output data in 2016 (stage 2)**

| Criteria | Abadan | Bandarabas | Arak | Esfehan | Tehran | Tabriz | Shiraz | Lavan | Kermanshah |
|---|---|---|---|---|---|---|---|---|---|
| $x_1^2$ | 0.0016 | 0.0016 | 0.0016 | 0.5752 | 0.0441 | 0.0964 | 0.1748 | 0.0016 | 0.1029 |
| $x_2^2$ | 0.3929 | 0.0008 | 0.0032 | 0.2838 | 0.0089 | 0.0946 | 0.1447 | 0.0008 | 0.0703 |
| $z_1^2$ | 0.2946 | 0.1123 | 0.0734 | 0.1668 | 0.1321 | 0.0505 | 0.0104 | 0.1559 | 0.0039 |
| $z_2^2$ | 0.2551 | 0.2034 | 0.1274 | 0.1598 | 0.1125 | 0.0471 | 0.0478 | 0.0293 | 0.0177 |
| $z_3^2$ | 0.1619 | 0.2272 | 0.1827 | 0.1845 | 0.1129 | 0.0540 | 0.0431 | 0.0195 | 0.0142 |
| $z_4^2$ | 0.3111 | 0.1197 | 0.1347 | 0.1682 | 0.1531 | 0.0326 | 0.0412 | 0.0282 | 0.0112 |
| $z_5^2$ | 0.2440 | 0.1883 | 0.1346 | 0.1588 | 0.1313 | 0.0666 | 0.0108 | 0.0340 | 0.0177 |
| $z_6^2$ | 0.2098 | 0.1896 | 0.1588 | 0.1687 | 0.1393 | 0.0759 | 0.0320 | 0.0157 | 0.0103 |
| $z_7^2$ | 0.1275 | 0.1372 | 0.2782 | 0.1765 | 0.1712 | 0.0834 | 0.0260 | 0.0001 | 0.0001 |
| $y_1^2$ | 0.1368 | 0.1361 | 0.2752 | 0.2010 | 0.1520 | 0.0588 | 0.0281 | 0.0035 | 0.0086 |
| $y_2^2$ | 0.2205 | 0.1443 | 0.2280 | 0.1652 | 0.1058 | 0.0577 | 0.345 | 0.0307 | 0.0132 |
| $y_3^2$ | 0.1951 | 0.1885 | 0.0655 | 0.0994 | 0.2677 | 0.1093 | 0.0109 | 0.0001 | 0.0635 |
| $y_4^2$ | 0.1864 | 0.1808 | 0.1232 | 0.2307 | 0.1302 | 0.0672 | 0.0347 | 0.0364 | 0.0105 |
| $y_5^2$ | 0.3019 | 0.2174 | 0.0829 | 0.1589 | 0.1158 | 0.0523 | 0.0250 | 0.0295 | 0.0165 |

Where the inputs are:
$x_1^1$: The consumption Oil (stage 1)
$x_2^1$: The consumption Fuel (stage 1)
$x_3^1$: Actual capacity (stage 1)
$x_4^1$: Complexity index (stage 1)
$x_5^1$: The number of human resources (stage 1)
$x_1^2$: The consumption super Gasoline (stage 2)
$x_2^2$: The consumption MTBE (stage 2)
The intermediate inputs/outputs are:
$z_1^1$: LPG (stage 1)
$z_2^1$: Light naphtha (stage 1)
$z_3^1$: Heavy naphtha (stage 1)

$z_4^1$: Kerosene (stage 1)
$z_5^1$: Gas oil (stage 1)
$z_6^1$: The residue of the distillation unit (stage 1)
$z_7^1$: Heavy gas oil (stage 1)
$z_1^2$: LPG (stage 2)
$z_2^2$: Light naphtha (stage 2)
$z_3^2$: Heavy naphtha (stage 2)
$z_4^2$: Kerosene (stage 2)
$z_5^2$: Gas oil (stage 2)
$z_6^2$: The residue of the distillation unit (stage 2)
$z_7^2$: Heavy gas oil (stage 2)
And the outputs are:
$y_1^1$: Crude oil (stage 1)
$y_2^1$: VB (stage 1)
$y_1^2$: LPG (stage 2)
$y_2^2$: Gasoline (stage 2)
$y_3^2$: Kerosene (stage 2)
$y_4^2$: Gas oil (stage 2)
$y_5^2$: Fuel oil (stage 2)

**Table 5.** **Total efficiencies in 2015 and 2016**

| system | $E_o$ (2015) | $D_I^G$ (2015) | $E_o$ (2016 ) | $D_I^G$ (2016) |
|---|---|---|---|---|
| Abadan | 0.7660 | 0.6068 | 0.5384 | 0.6071 |
| Bandarabas | 0.7593 | 0.7602 | 0.8485 | 0.7482 |
| Arak | 0.6170 | 0.6430 | 0.8444 | 0.5911 |
| Esfehan | 0.7810 | 0.7726 | 0.6061 | 0.8014 |
| Tehran | 0.9048 | 0.9048 | 0.8792 | 0.8934 |
| Tabriz | 0.7528 | 0.6528 | 0.6744 | 0.6086 |
| Shiraz | 0.4779 | 0.8832 | 0.4350 | 0.8717 |
| Lavan | 0.4231 | 0.4991 | 0.5892 | 0.5008 |
| Kermanshah | 0.5016 | 0.5005 | 0.5493 | 0.5018 |

**Table 6.** **First stage efficiencies in 2015 and 2016**

| system | $E_1$ (2015) | $\theta_1$ (2015) | $E_1$ (2016 ) | $\theta_1$ (2016) |
|---|---|---|---|---|
| Abadan | 0.9171 | 0.2136 | 0.5814 | 0.2143 |
| Bandarabas | 0.9068 | 0.5204 | 0.9048 | 0.4965 |
| Arak | 0.7967 | 0.3898 | 0.8296 | 0.1823 |
| Esfehan | 0.9822 | 0.9641 | 0.8333 | 1.0000 |
| Tehran | 0.9228 | 1.0000 | 0.7173 | 1.0000 |
| Tabriz | 0.8071 | 0.5997 | 0.5904 | 0.5768 |
| Shiraz | 0.9409 | 1.0000 | 0.6933 | 0.9928 |
| Lavan | 0.7341 | 0.0032 | 0.4038 | 0.0042 |
| Kermanshah | 0.9350 | 0.0052 | 0.6132 | 0.0056 |

**Table 7.** **Second stage efficiencies in 2015 and 2016**

| system | $E_2$ (2015) | $\theta_2$ (2015) | $E_2$ (2016 ) | $\theta_2$(2016) |
|---|---|---|---|---|
| Abadan | 0.8219 | 1.0000 | 0.6793 | 1.0000 |
| Bandarabas | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| Arak | 0.6284 | 0.8962 | 0.6754 | 1.0000 |
| Esfehan | 0.6771 | 0.5810 | 0.5411 | 0.6028 |
| Tehran | 0.9683 | 0.6709 | 0.8079 | 0.7869 |
| Tabriz | 0.7765 | 0.7050 | 0.7785 | 0.6404 |
| Shiraz | 0.3935 | 0.7663 | 0.2758 | 0.7506 |
| Lavan | 0.4280 | 0.9950 | 0.8755 | 0.9974 |
| Kermanshah | 0.6887 | 0.9959 | 0.6579 | 0.9981 |

**Table 8.** **Malmquist productivity indexes (2015-2016)**

| system | $MPI_1$ | $MPI_2$ | $MPI_o$ |
|---|---|---|---|
| Abadan | 0.9166 | 0.8267 | 0.4615 |
| Bandarabas | 5.2070 | 1.0879 | 6.4568 |
| Arak | 0.3291 | 1.0871 | 0.4569 |
| Esfehan | 0.9153 | 1.0168 | 0.8967 |
| Tehran | 0.8781 | 0.2609 | 0.8641 |
| Tabriz | 0.7600 | 2.4028 | 0.8940 |
| Shiraz | 0.9886 | 1.1749 | 0.9287 |
| Lavan | 0.6562 | 1.6727 | 1.6317 |
| Kermanshah | 1.0501 | 1.3536 | 1.6583 |

**Table 9.** **Malmquist productivity indexes (2015-2016)**

| system | $MPI_I^G(p_1^t, p_1^{t+1})$ | $MPI_I^G(p_2^t, p_2^{t+1})$ | $MPI_I^G(o^t, o^{t+1})$ |
|---|---|---|---|
| Abadan | 1.0033 | 1.0000 | 1.0005 |
| Bandarabas | 0.9541 | 1.0000 | 0.9842 |
| Arak | 0.4677 | 1.1158 | 0.9193 |
| Esfehan | 1.0372 | 1.0375 | 1.0373 |
| Tehran | 1.0000 | 1.1729 | 1.0694 |
| Tabriz | 0.9618 | 0.9084 | 0.9323 |
| Shiraz | 0.9928 | 0.9795 | 0.9870 |
| Lavan | 1.0769 | 1.0024 | 1.0034 |
| Kermanshah | 1.0769 | 1.0022 | 1.0024 |

In Tables (5) to (7), $E_o(t)$, $E_1(t)$, and $E_2(t)$ represent the efficiency scores of the system o, stage 1, and stage 2 in period t ($t \in \{2015, 2016\}$) respectively, obtained by Tavana et al.'s model and $D^G_{I\ o}(t)$, $\theta_1(t)$, and $\theta_2(t)$ represent the efficiency scores of the system o, stage 1, and stage 2 in period t ($t \in \{2015, 2016\}$) respectively, obtained by the model (3.1). The results show that the efficiency scores calculated by the proposed model are close to the results of Tavana et al.'s model. The slight difference in scores is due to the difference in the production frontier of models. MPIs obtained from the proposed model show that significant changes in productivity have not been achieved from 2015 to 2016 and indeed, minor changes in productivity can be seen. The highest productivity increase is related to the Tehran refinery with a value of 1.0694 and the lowest is related to Abadan refinery with a value of 1.0005. Also, the highest productivity decline is related to Tabriz refinery with a value of 0.9323 and the lowest is linked to Shiraz refinery with a value of 0.9870. The difference between the productivity changes resulting from the proposed model and Tavana et al.'s model is due to a special version of the $MPI$ designed in their model. Also, the frontier considered by them has been defined for each stage of production, and separate for each period, while in the proposed model of this study, the global frontier has been considered.

## 4.2. Structural example

Here, we are considering a simple structural example where undesirable and non-discretionary data are present in three-stage series network systems. We also explain how the model (3.1) is used to calculate the $MPI$s of the system and divisions. Consider 10 systems $1, 2, \ldots, 10$ include three divisions with the structure shown in Figure (2) and the data shown in Tables (10) and (11) at three periods $t$, $t+1$, and $t+2$.

**Table 10.** **Data for the general series structure example.**

| period | $x_1^d$ | $x_2^d$ | $x_3^{ud}$ | $x_4^{nd}$ | $z_1^d$ | $y_1^d$ | $y_2^d$ | $y_3^{ud}$ | $y_4^{nd}$ |
|---|---|---|---|---|---|---|---|---|---|
| system 1 | | | | | | | | | |
| $t$ | (2 3 2) | (5 5 4) | (3 4 5) | (5 5 1) | (3 2 2 2) | (5 6 6) | (2 4 4) | (6 5 4) | (5 4 3) |
| $t+1$ | (2 2 1) | (4 2 2) | (3 2 1) | (5 4 3) | (4 4 4 4) | (4 5 4) | (2 1 1) | (6 6 6) | (5 5 5) |
| $t+2$ | (3 2 3) | (4 3 5) | (2 3 5) | (6 5 6) | (2 2 3 2) | (3 2 2) | (4 3 3) | (5 4 4) | (4 4 4) |
| system 2 | | | | | | | | | |
| $t$ | (3 4 4) | (5 5 6) | (3 4 6) | (3 3 2) | (3 3 4 1) | (5 5 6) | (2 2 2) | (4 4 3) | (6 5 6) |
| $t+1$ | (2 2 5) | (5 2 6) | (3 2 8) | (4 4 3) | (3 2 3 2) | (5 4 4) | (3 3 3) | (5 4 4) | (5 4 4) |
| $t+2$ | (3 4 3) | (6 5 8) | (2 2 5) | (3 3 2) | (4 4 4 4) | (6 7 6) | (3 2 2) | (5 5 5) | (5 3 5) |
| system 3 | | | | | | | | | |
| $t$ | (4 5 3) | (6 5 6) | (3 4 1) | (4 5 6) | (2 1 2 2) | (7 6 6) | (3 4 4) | (6 6 5) | (5 4 5) |
| $t+1$ | (3 3 3) | (5 4 3) | (4 4 6) | (5 4 5) | (3 2 3 1) | (7 8 8) | (4 5 4) | (6 4 5) | (4 4 4) |
| $t+2$ | (2 2 3) | (4 4 4) | (4 5 8) | (4 5 6) | (2 2 4 2) | (6 5 4) | (3 2 4) | (6 7 7) | (4 3 2) |
| system 4 | | | | | | | | | |
| $t$ | (4 5 4) | (6 5 6) | (4 4 2) | (5 4 4) | (4 3 4 3) | (6 4 6) | (4 3 4) | (4 5 4) | (8 8 8) |
| $t+1$ | (4 4 4) | (6 7 8) | (3 3 2) | (5 3 3) | (3 2 5 4) | (8 7 5) | (5 6 5) | (4 3 2) | (7 6 7) |
| $t+2$ | (3 2 1) | (7 3 4) | (3 3 1) | (3 2 4) | (4 4 5 5) | (7 5 6) | (5 6 6) | (5 5 6) | (7 6 8) |
| system 5 | | | | | | | | | |
| $t$ | (5 4 4) | (5 5 4) | (4 4 5) | (3 3 4) | (3 2 3 3) | (6 5 5) | (6 7 8) | (6 6 5) | (5 4 4) |
| $t+1$ | (4 3 3) | (6 5 8) | (5 6 5) | (5 4 3) | (4 4 3 3) | (5 4 4) | (6 5 4) | (6 6 6) | (6 5 5) |
| $t+2$ | (4 5 5) | (7 4 7) | (4 3 2) | (4 4 5) | (5 5 6 6) | (5 3 4) | (5 4 3) | (7 6 5) | (9 8 8) |

**Table 11.** **Data for the general series structure example-Continued.**

| period | $x_1^d$ | $x_2^d$ | $x_3^{ud}$ | $x_4^{nd}$ | $z_1^d$ | $y_1^d$ | $y_2^d$ | $y_3^{ud}$ | $y_4^{nd}$ |
|---|---|---|---|---|---|---|---|---|---|
| system 6 | | | | | | | | | |
| $t$ | (5 3 4) | (7 6 8) | (5 6 4) | (3 2 2) | (4 4 5 5) | (6 6 6) | (3 3 2) | (8 8 8) | (3 2 2) |
| $t+1$ | (5 4 4) | (8 5 9) | (4 5 7) | (3 2 2) | (4 3 4 3) | (6 7 7) | (4 3 3) | (8 7 7) | (2 1 2) |
| $t+2$ | (8 7 8) | (7 6 8) | (4 3 4) | (3 1 1) | (5 4 5 5) | (7 8 9) | (4 3 3) | (7 6 7) | (4 5 5) |
| system 7 | | | | | | | | | |
| $t$ | (5 6 5) | (6 5 4) | (5 6 4) | (4 3 3) | (5 3 4 3) | (6 5 5) | (4 4 4) | (8 7 8) | (7 7 8) |
| $t+1$ | (6 5 4) | (5 4 6) | (5 5 4) | (5 4 4) | (6 6 6 6) | (8 7 8) | (3 3 3) | (8 9 8) | (8 6 8) |
| $t+2$ | (6 7 8) | (5 6 5) | (6 6 5) | (5 5 4) | (5 2 6 3) | (8 8 6) | (3 3 2) | (9 8 8) | (7 5 5) |
| system 8 | | | | | | | | | |
| $t$ | (6 5 6) | (5 4 6) | (6 5 6) | (4 3 3) | (5 4 4 4) | (8 9 9) | (6 6 7) | (9 8 9) | (5 4 5) |
| $t+1$ | (7 6 8) | (4 6 3) | (5 5 4) | (3 2 2) | (6 5 6 6) | (7 8 7) | (5 4 4) | (8 8 8) | (5 6 5) |
| $t+2$ | (7 7 9) | (4 5 8) | (4 5 6) | (3 3 1) | (8 7 7 7) | (7 6 6) | (6 7 7) | (7 6 6) | (4 5 6) |
| system 9 | | | | | | | | | |
| $t$ | (5 4 8) | (6 2 3) | (3 3 2) | (3 4 4) | (6 4 7 4) | (8 8 8) | (6 5 5) | (8 7 8) | (4 5 9) |
| $t+1$ | (6 3 7) | (7 3 4) | (4 4 6) | (4 3 3) | (7 5 6 5) | (9 9 8) | (6 5 6) | (7 6 5) | (5 4 5) |
| $t+2$ | (7 5 8) | (7 9 8) | (4 3 5) | (5 4 4) | (5 4 4 3) | (8 7 6) | (7 8 9) | (8 8 8) | (6 5 6) |
| system 10 | | | | | | | | | |
| $t$ | (7 8 9) | (8 5 6) | (6 6 5) | (5 5 4) | (5 3 4 3) | (9 9 8) | (8 8 9) | (8 7 8) | (6 4 6) |
| $t+1$ | (7 5 4) | (7 4 5) | (5 4 3) | (5 4 4) | (6 5 6 5) | (9 9 8) | (8 8 7) | (9 8 7) | (7 6 7) |
| $t+2$ | (8 3 4) | (8 4 6) | (4 5 4) | (4 5 5) | (7 6 6 6) | (9 4 5) | (7 6 8) | (7 6 5) | (7 6 7) |

It is reminded that in the inputs and outputs, the numbers in parentheses correspond to the first, second, and third stages of the systems, respectively. In intermediate products, the numbers in parentheses are the output of the first stage, the input of the second stage, the output of the second stage, and the input of the third stage, respectively. The first two inputs are assumed to be desirable ($x_i^d, i = 1, 2.$), the third input is undesirable($x_3^{ud}$), and the fourth input is assumed to be non-discretionary ($x_4^{nd}$)in all divisions of the systems. Every system has an intermediate product. Also, the first two outputs are assumed to be desirable($y_i^d, i = 1, 2.$), the third output is undesirable ($y_3^{ud}$), and the fourth output is assumed to be non-discretionary ($y_4^{nd}$) in all divisions of the systems. Model (3.1) is applied to data of Tables (10) and (11) with the results reported in Tables 15 to 17.

**Table 12.** **results obtained from model (3.1) at period t**

| system | $\theta_1^t$ | $\theta_2^t$ | $\theta_3^t$ | $(D_I^G)^t$ |
|---|---|---|---|---|
| 1 | 1.00000 | 0.88235 | 1.00000 | 0.96078 |
| 2 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 3 | 0.87118 | 0.72199 | 0.71766 | 0.77076 |
| 4 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 5 | 1.00000 | 0.92105 | 1.00000 | 0.97368 |
| 6 | 1.00000 | 1.00000 | 0.50000 | 0.83333 |
| 7 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 8 | 1.00000 | 1.00000 | 0.89394 | 0.96456 |
| 9 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 10 | 1.00000 | 1.00000 | 0.96529 | 0.98843 |

**Table 13.** results obtained from model (3.1) at period t+1

| system | $\theta_1^{t+1}$ | $\theta_2^{t+1}$ | $\theta_3^{t+1}$ | $(D_I^G)^{t+1}$ |
|---|---|---|---|---|
| 1 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 2 | 1.00000 | 0.95652 | 1.00000 | 0.98551 |
| 3 | 0.98220 | 1.00000 | 1.00000 | 0.99407 |
| 4 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 5 | 1.00000 | 1.00000 | 0.80114 | 0.93371 |
| 6 | 0.88796 | 1.00000 | 1.00000 | 0.96265 |
| 7 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 8 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 9 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 10 | 1.00000 | 1.00000 | 0.96566 | 0.98855 |

**Table 14.** results obtained from model (3.1) at period t+2

| system | $\theta_1^{t+2}$ | $\theta_2^{t+2}$ | $\theta_3^{t+2}$ | $(D_I^G)^{t+2}$ |
|---|---|---|---|---|
| 1 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 2 | 0.97900 | 0.88783 | 0.86863 | 0.91182 |
| 3 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 4 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 5 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 6 | 0.95238 | 1.00000 | 1.00000 | 0.98413 |
| 7 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 8 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 9 | 0.88054 | 0.82291 | 0.77610 | 0.82652 |
| 10 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |

**Table 15.** $MPI_I^G$s of divisions and systems from period t to t+1.

| system | $MPI_I^G(p_1^t, p_1^{t+1})$ | $MPI_I^G(p_2^t, p_2^{t+1})$ | $MPI_I^G(p_3^t, p_3^{t+1})$ | $MPI_I^G(o^t, o^{t+1})$ |
|---|---|---|---|---|
| 1 | 1.00000 | 0.99303 | 1.36169 | 1.09459 |
| 2 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 3 | 0.88476 | 0.72414 | 0.71766 | 0.77520 |
| 4 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 5 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 6 | 1.12618 | 1.00000 | 0.50000 | 0.86566 |
| 7 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 8 | 1.00000 | 1.00000 | 0.91564 | 0.97188 |
| 9 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 10 | 1.00000 | 1.00000 | 1.02471 | 1.00807 |

Table (15) shows productivity changes of the systems and divisions during periods $t$ to $t+1$. According to the results, systems 1 and 10 have progressed during periods $t$ to $t+1$. Systems 3 , 6, and 8 have regressed. The remaining systems have not changed productivity. Despite the regression of the second division of system 1, this system has improved its performance. By improving this division, the progress of

**Table 16.** $MPI_I^G$**s of divisions and systems from period t+1 to t+2.**

| system | $MPI_I^G(p_1^{t+1}, p_1^{t+2})$ | $MPI_I^G(p_2^{t+1}, p_2^{t+2})$ | $MPI_I^G(p_3^{t+1}, p_3^{t+2})$ | $MPI_I^G(o^{t+1}, o^{t+2})$ |
|---|---|---|---|---|
| 1 | 1.00000 | 1.00000 | 0.73438 | 0.91146 |
| 2 | 1.02145 | 1.12634 | 1.15123 | 1.09670 |
| 3 | 0.99150 | 1.00000 | 1.00000 | 0.99717 |
| 4 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 5 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 6 | 0.91136 | 1.00000 | 1.00000 | 0.97817 |
| 7 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 8 | 1.00000 | 1.00000 | 0.91564 | 0.97188 |
| 9 | 1.13567 | 1.21520 | 1.28849 | 1.20989 |
| 10 | 1.00000 | 1.00000 | 0.96566 | 0.98855 |

**Table 17.** $MPI_I^G$**s of divisions and systems from period t to t+2.**

| system | $MPI_I^G(p_1^{t}, p_1^{t+2})$ | $MPI_I^G(p_2^{t}, p_2^{t+2})$ | $MPI_I^G(p_3^{t}, p_3^{t+2})$ | $MPI_I^G(o^{t}, o^{t+2})$ |
|---|---|---|---|---|
| 1 | 1.00000 | 0.99303 | 1.00000 | 0.99768 |
| 2 | 1.02145 | 1.12634 | 1.15123 | 1.09670 |
| 3 | 0.87724 | 0.72414 | 0.71766 | 0.77301 |
| 4 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 5 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 6 | 1.05000 | 1.00000 | 0.50000 | 0.84677 |
| 7 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 8 | 1.00000 | 1.00000 | 0.91564 | 0.97188 |
| 9 | 1.13567 | 1.21520 | 1.28849 | 1.20989 |
| 10 | 1.00000 | 1.00000 | 0.98952 | 0.99653 |

system 1 can be increased. As an example of regressed systems, we can refer to system 6. We can see from table (15) that its third division has caused the system to regress. The system can be improved by improving this division.
Table (16) shows productivity changes of the systems and divisions during periods $t+1$ to $t+2$. Systems 2 and 9 progressed from t+1 to t+2. Systems 1, 3, 6, 8, and 10 regressed. The other systems had no productivity changes. As an example of regressed systems, consider a system 10. We can see from table (16) that its third division caused the regression. The system can be improved by improving this division.

Table (17) shows productivity changes of the systems and divisions during periods $t$ to $t+2$. For example, system 2 has had progress during periods $t$ to $t+2$ and the average for it is greater than one. System 9 also holds in this situation, but the others are not like this. We can see the effect of the divisions on the productivity changes of the systems. The Malmquist index of the entire system equals the weighted combination of those values for the divisions. The weights are considered for model (3.1). If one division regresses during periods $t$ to $t+2$ and the rest stays unchanged, the entire system will regress. System 8 is an example of this. The first two divisions of this system have remained unchanged. The third division has regressed. This has caused System 8 to regress. According to the results, System 2 experienced the worst conditions during periods $t$ to $t+2$, and had the greatest regression among 10 systems. Meanwhile, System 9 experienced the ideal conditions and had the most appropriate progress.

## 5. Conclusion

In this article we evaluated multi-stage series systems with the new model. The systems were evaluated in the presence of undesirable and non-discretionary data. The proposed model of this study simultaneously calculated and presented the productivity changes of systems and divisions during period $T$. It used the input-oriented global Malmquist productivity index. A separate parameter was defined for each division's input in each period. The proposed model has a main advantage. It detects the effect of divisions on system productivity changes. The Malmquist index of the whole system was considered to be equal to the weighted combination of those values for the divisions. The weights were considered in the model (3.1). In real-world issues, we sometimes encounter systems with undesirable and non-discretionary data, such as health systems and waste recycling systems. Thus, studying and evaluating them is very necessary and unavoidable. The mentioned systems can be investigated and improved. We can identify the defective divisions with the model presented in this study. To check the ability of the proposed model, we presented examples in section 4. In the first example the results obtained from the proposed and Tawana et. al's model were compared . It was observed that the proposed model could evaluate a multi-stage series systems in different periods. We designed the second example as a structural example. It defined hypothetical systems with undesirable and non-discretionary data. The proposed model solved this example and evaluated the mentioned systems . We also examined and interpreted the effect of productivity changes in the division. As a result, we provided the necessary recommendations to improve the regressive divisions. We aimed to improve the status of the regressive systems. Future research can explore related applications with other types of data such as interval, stochastic, and fuzzy data in general multi-stage systems.

**Conflict of Interest:** The authors declare that they have no conflict of interest.

**Data Availability:** Data sharing is not applicable to this article as no data sets were generated or analyzed during the current study.

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