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Research Article

Strength based domination in graphs

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Abstract: Let $G = (V, E)$ be a connected graph. Let $A \subseteq V$ and $v \in V - A$. The dominating strength of A on v is defined by $s(v, A) = \sum_{u \in A} \frac{1}{d(u,v)}$. A subset D of V is called a strength based dominating set if for every vertex $v \notin D$, there exists a subset A of D such that $s(v, A) \geq 1$. The sb-domination number $\gamma_{sb}(G)$ is the minimum cardinality of a strength based dominating set of G. In this paper we initiate a study of this parameter and indicate directions for further research.

Keywords: distance, domination, dominating strength, sb-domination.

AMS Subject classification: 05C69

1. Introduction

By a graph $G = (V, E)$ we mean a finite, undirected, connected graph with neither loops nor multiple edges. For graph theoretic terminology we refer to the book [1]. A subset D of V is called a dominating set of G if every vertex v in $V - D$ is adjacent to a vertex u in D. The minimum cardinality of a dominating set of G is called the domination number of G and is denoted by $\gamma(G)$. The concept of domination in graphs and its several variants have been extensively investigated. For fundamentals of domination in graphs we refer to [5].

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Different types of dominating sets have been formulated by putting restrictions on the induced subgraph $G[D]$. Connected domination, total domination, independent domination and paired domination are some of the domination parameters under this category. For a detailed study of total domination in graphs we refer to the book [6]. Another type of generalization is by putting restrictions on $N(v) \cap D$ and examples of such a type are weak domination, strong domination, k-domination and perfect domination. For further details of various types of domination models we refer to the Appendix in [5].

The distance $d(u, v)$ between two vertices u and v in a graph is the length of a shortest u-v path in G. The eccentricity of a vertex v is defined by $ecc(v) = \max\{d(u, v) : u \in V\}$. The radius and diameter of G are defined by $\text{rad}(G) = \min\{ecc(v) : v \in V\}$ and diam $(G) = \max\{ecc(v) : v \in V\}$.

In several real life situations such as social networks, communication networks and biological networks, the influence of a vertex extends beyond its neighborhood but decreases with distance. To address this problem Dankelmann et al. [2] introduced the concept of exponential domination and exponential domination number of a graph, in which the dominating power of a vertex is decreasing exponentially by the factor $\frac{1}{2}$ with distance.

Goddard et al. [4] introduced the concept of disjunctive domination number of a graph. This concept reconsider in [3].

Definition 1. Let $G = (V, E)$ be a connected graph. A subset D of V is called a disjunctive dominating set of G, if every vertex $v \notin D$ is adjacent to a vertex in D or has at least two distinct vertices at a distance two from v. The minimum cardinality of a disjunctive dominating set of G is called disjunctive domination number of G and is denoted by $\gamma_2^d(G)$.

Theorem 1. *([4]) Let G be any graph. Then $\gamma_2^d(G) \leq \gamma(G)$.*

Theorem 2. *([4]) For any positive integer n, $\gamma_2^d(P_n) = \left\lceil \frac{n+1}{4} \right\rceil$.*

Theorem 3. *([4]) For any positive integer $n \geq 3$,*

$$\gamma_2^d(C_n) = \begin{cases} 2 & \text{if } n = 4 \\ \lceil \frac{n}{4} \rceil & \text{if } n \neq 4 \end{cases}$$

We need the following definitions and theorem.

Definition 2. Let G_1 and G_2 be two graphs. The corona $G_1 \circ G_2$ is the graph obtained from one copy of G_1 and $|V(G_1)|$ copies of G_2 by joining the i^{th} vertex of G_1 to all the vertices in the i^{th} copy of G_2.

Definition 3. The Cartesian product $G = G_1 \Box G_2$ of two graphs G_1 and G_2 has $V(G) = V(G_1) \times V(G_2)$ and two vertices (u_1, v_1) and (u_2, v_2) are adjcent in G if either $u_1 = u_2$ and $v_1 v_2 \in E(G_2)$ or $v_1 = v_2$ and $u_1 u_2 \in E(G_1)$.

Theorem 4. *([5], Page 56) For any connected graph G, $\left\lceil \frac{diam(G)+1}{3} \right\rceil \leq \gamma(G)$.*

In this paper we introduce the concept of strength based domination in graphs which is a variant of the exponential model considered by Dankelmann et al. [2]. We present several basic results on strength based domination number and indicate directions for further research.

2. On sb-domination

In a social network a member v is very often influenced by another member u who is not in its neighborhood $N(v)$. In fact there is a possibility that the member v is influenced by a group of members who are not in $N(v)$. We propose the concept of dominating strength and the associated parameters to address the above situation.

Definition 4. Let $G = (V, E)$ be a connected graph and let $u, v \in V$. The dominating strength $s(u, v)$ between u and v is defined as $s(u, v) = \frac{1}{d(u,v)}$. The dominating strength $\text{ds}(v)$ of v is defined as $\text{ds}(v) = \sum_{u \neq v} s(u, v) = \sum_{u \neq v} \frac{1}{d(u,v)}$. The sequence $\Pi = (\text{ds}(v_1), \text{ds}(v_2), \ldots, \text{ds}(v_n))$ where $\text{ds}(v_1) \geq \text{ds}(v_2) \geq \cdots \geq \text{ds}(v_n)$ is called the dominating strength sequence or simply the ds-sequence of G.

Example 1. For the graph G given in Figure 1,

$$\text{ds}(v_i) = \begin{cases} \frac{157}{60} & \text{if } i = 1 \text{ or } 6 \\ \frac{43}{12} & \text{if } i = 2 \text{ or } 5 \\ \frac{13}{3} & \text{if } i = 3 \text{ or } 4 \\ \frac{11}{3} & \text{if } i = 7. \end{cases}$$

Hence the ds-sequence Π is given by

$$\begin{aligned} \Pi &= \left(\frac{13}{3}, \frac{13}{3}, \frac{11}{3}, \frac{43}{12}, \frac{43}{12}, \frac{157}{60}, \frac{157}{60} \right) \\ &= (\text{ds}(v_3), \text{ds}(v_4), \text{ds}(v_7), \text{ds}(v_2), \text{ds}(v_5), \text{ds}(v_1), \text{ds}(v_6)). \end{aligned}$$

Observation 5. Let G be a connected graph of order n and let $v \in V$. Then $deg(v) \leq \text{ds}(v)$ and equality holds if and only if $deg(v) = n - 1$.

Definition 5. Let G be a connected graph of order n. Let $A \subseteq V$ and $v \in V - A$. Then the dominating strength of A on v is defined by $\text{ds}(A, v) = \sum_{u \in A} \frac{1}{d(v,u)}$.

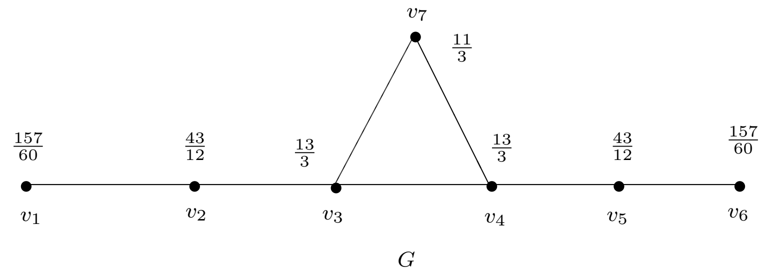

Figure 1. **A graph and its ds-sequence**

Example 2. A graph G with its ds-sequence is given in Figure 1. In this graph

$$\mathrm{ds}(\{v_1, v_2\}, v_6) = \frac{1}{d(v_6, v_1)} + \frac{1}{d(v_6, v_2)} = \frac{1}{5} + \frac{1}{4} = 0.45.$$

If D is a dominating set of G, then any vertex $v \notin D$ is dominated by a vertex in D. In the case of disjunctive domination, v is dominated by a single vertex in D or is dominated by a set of two vertices in D each at distance 2 from v. We now introduce the concept of strength based domination in which v is dominated by a subset D_1 of D and this is a generalization of domination and disjunctive domination.

Definition 6. Let $G = (V, E)$ be a connected graph. A subset D of V is called a strength based dominating set or a sb-dominating set of G if for every $v \in V - D$, there exists a subset D_1 of D such that $\mathrm{ds}(D_1, v) \geq 1$. The minimum cardinality of a sb-dominating set of G is called the sb-domination number of G and is denoted by $\gamma_{sb}(G)$. Also sb-dominating set of cardinality γ_{sb} is called a γ_{sb}-set of G.

Observation 6. Let D be a sb-dominating set of G and let $v \in V - D$. Obviously $\mathrm{ds}(D, v) \geq 1$ if and only if there exists a subset D_1 of D such that $\mathrm{ds}(D_1, v) \geq 1$. The concept of sb-domination has interesting applications in social networks and in a large network identifying a subset D_1 such that the members of D_1 can collectively influence a member $v \in V - D$ is a significant and relevant issue. Thus from application perspective the subset D_1 in the above definition plays a crucial role.

Observation 7. For any graph G, we have $\gamma(G) = \gamma_{sb}(G) = 1$ if and only if $\Delta = n - 1$.

Observation 8. Clearly any dominating set of G and any disjunctive dominating set of G are sb-dominating sets. Hence $\gamma_{sb}(G) \leq \gamma_2^d(G) \leq \gamma(G)$.

Example 3. For the Petersen graph G given in Figure 2, $D = \{v_1, v_2\}$ is a sb-dominating set of G, since $d(u, v_1) = d(u, v_2) = 2$ for all vertices $u \in (V - D) \cup N(v_1) \cup N(v_2)$. Also $\Delta = 3 < n - 1$. Hence $\gamma_{sb}(G) = 2$.

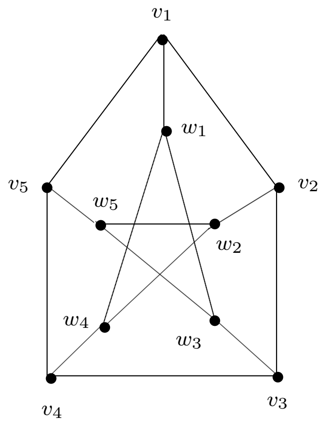

Figure 2. **Petersen graph** $G : \gamma_{sb}(G) = 2$.

Example 4. Let $G = K_n \circ K_1$. Let $V(K_n) = \{v_1, v_2, \ldots, v_n\}$ and let w_i be the pendent vertex adjacent to v_i. Clearly $D = \{v_1, v_2\}$ is a *sb*-dominating set of G and hence $\gamma_{sb}(G) \leq 2$. Since $\Delta < |V(G)| - 1, \gamma_{sb}(G) \geq 2$. Hence $\gamma_{sb}(G) = 2$. Since $\gamma(G) = n$, it follows that the difference between $\gamma_{sb}(G)$ and $\gamma(G)$ can be arbitrarily large.

Theorem 9. *For any positive integer* k*, there exists a graph* G *with* $\gamma_{sb}(G) = k$.

Proof. For any graph G with $\gamma(G) = 1$ or 2, we have $\gamma_{sb}(G) = \gamma(G)$.
Suppose $k \geq 3$. Let G be the graph obtained from $K_{1,k}$ by sub-dividing each edge $(k-2)$ times. Let $V(K_{1,k}) = \{v_0, v_1, \ldots, v_k\}$ and $deg(v_0) = k$. Let $w_{i_1}, w_{i_2}, \ldots, w_{i_{(k-2)}}$) be the vertices sub-dividing the edge v_0v_i. Let $P_i = (v_0, w_{i_1}, w_{i_2}, \ldots, w_{i_{(k-2)}}, v_i)$. Let $D = \{w_{11}, w_{21}, \ldots, w_{k1}\}$. Now, $\text{ds}(D, v_i) = (k-1)\frac{1}{k} + \frac{1}{k-2} = 1 + \left(\frac{1}{k-2} - \frac{1}{k}\right) > 1$. Also, $\text{ds}(D, u) \geq \text{ds}(D, v_i)$ for all $u \in V - D$. Hence D is a *sb*-dominating set of G and therefore $\gamma_{sb}(G) \leq k$. Now let S be any γ_{sb}-set of G. If $S \cap V(P_i) = \emptyset$ for some i, then $\text{ds}(v_i, S) \leq \frac{k-1}{k} + \frac{1}{k-1} < 1$. Hence $S \cap V(P_i) \neq \emptyset$ for all $i, 1 \leq i \leq k$ and so $|S| \geq k$. Thus $\gamma_{sb}(G) \geq k$ and hence $\gamma_{sb}(G) = k$. □

3. Bounds for γ_{sb}

The following theorems give an upper bound for $\gamma_{sb}(G)$ and a characterization of all extremal graphs which attain the bound.

Theorem 10. *Let* G *be a connected graph of order* n*. Let* $\Delta_{sb} = \max\{\text{ds}(v) : v \in V\}$. *Then* $\gamma_{sb}(G) \leq n - \lfloor \Delta_{sb} \rfloor$.

Proof. Let $v \in V$ and $\text{ds}(v) = \Delta_{sb}$. Let $n - i = \lfloor \Delta_{sb} \rfloor$. Hence $n - i \leq \Delta_{sb} < n - i + 1$. Therefore, $n - i \leq \text{ds}(v) < n - i + 1$. Also $\text{ds}(v) \leq deg(v) + \frac{n-1-deg(v)}{2} = \frac{deg(v)}{2} + \left(\frac{n-1}{2}\right)$

and hence $n - i \leq \frac{deg(v)}{2} + \left(\frac{n-1}{2}\right)$. Thus, $deg(v) \geq n - 2i + 1$.
Since $\lfloor \Delta_{sb} \rfloor = n - i$, we have $deg(v) \leq n - i$. Hence, $n - 2i + 1 \leq deg(v) \leq n - i$. Thus, $deg(v) = n - 2i + j$ where $1 \leq j \leq i$. Now let $G_1 = G[V - N[v]]$. Clearly, $|V(G_1)| = n - deg(v) - 1 = 2i - j - 1$. Let A denote the set of all isolated vertices in G_1 and let $|A| = k$. Then $G_1 - A$ is a graph of order $2i - j - 1 - k$ and has no isolated vertices. Let D be a γ-set of $G_1 - A$. Therefore

$$|D| = \gamma(G_1 - A) \leq \left\lfloor \frac{2i - j - 1 - k}{2} \right\rfloor \leq \left\lfloor \frac{2i - 2 - k}{2} \right\rfloor = i - 1 - \left\lceil \frac{k}{2} \right\rceil.$$

Hence

$$|D| \leq i - 1 - \left\lceil \frac{k}{2} \right\rceil. \tag{3.1}$$

If $k = 0$, let $B = \{v\}$.
If $k = 1$ or 2, let B be a subset of $N(v)$ of minimum order such that B dominates A. Clearly $|B| = 1$ if $k = 1$ and $|B| = 1$ or 2 if $k = 2$.
If $k \geq 3$, let $B = \{v, u, w\}$ where $u, w \in A$. Let $D_1 = D \cup B$. It follows from (3.1) that

$$|D_1| \leq i \quad \text{if } k = 0, 3 \text{ } or \text{ } 4 \text{ or } k = 2 \text{ and } |B| = 2 \tag{3.2}$$

$$\text{and } |D_1| < i \quad \text{if } k = 2 \text{ and } |B| = 1 \text{ or } k \geq 5. \tag{3.3}$$

Since D dominates $G_1 - A$ and B sb-dominates $N[v] \cup A$, it follows that D_1 is a sb-dominating of G. Also $|D_1| \leq i = n - \lfloor \Delta_{sb} \rfloor$. Hence $\gamma_{sb} \leq n - \lfloor \Delta_{sb} \rfloor$. □

Theorem 11. *Let $G = (V, E)$ be a connected graph of order n. Then $\gamma_{sb}(G) = n - \lfloor \Delta_{sb} \rfloor$ if and only if the following conditions hold.*

(i) There exists a vertex v such that $\mathrm{ds}(v) = \Delta_{sb}$ and $deg(v) = 2\lfloor \Delta_{sb} \rfloor + 1 - n$.

(ii) The number of isolated vertices k in $G - N[v]$ is at most 4 and if $k = 2$, then the two isolated vertices have no common neighbor in $N(v)$.

Proof. $\gamma_{sb} = n - \lfloor \Delta_{sb} \rfloor$ if and only if equality holds in (1) and (2) of Theorem 10. Also equality holds in (1) if and only if $j = 1$ and $\gamma(G - N[v]) = \left\lfloor \frac{|V(G) - N[v]|}{2} \right\rfloor + k$. Equality holds in (2) if and only if $k \leq 4$ and $|B| = 2$ when $k = 2$. Hence the result follows. □

We now proceed to obtain lower bounds for γ_{sb}.

Theorem 12. *Let G be a connected graph of order n. Then $\gamma_{sb}(G) \geq \left\lceil \frac{n}{1+\Delta_{sb}} \right\rceil$ and the bound is sharp.*

Proof. Let D be a γ_{sb}-set of G. Since $\text{ds}(v) \leq \Delta_{sb}$ for all $v \in D$, we have $\sum_{v\in D} \text{ds}(v) \leq |D|\Delta_{sb}$. Also $\text{ds}(D,w) \geq 1$ for all $w \in V - D$ and hence $\sum_{v\in D} \text{ds}(v) \geq n - |D|$. Thus $n - |D| \leq \sum_{v\in D} \text{ds}(v) \leq |D|\Delta_{sb}$. Hence $n \leq |D|(\Delta_{sb} + 1)$ and so $\gamma_{sb} \geq \left\lceil \frac{n}{1+\Delta_{sb}} \right\rceil$. Also for the graph G given in Figure 3, $\Delta_{sb} = 5.75$ and $D = \{v, w\}$ is a γ_{sb}-set of G. Hence $\gamma_{sb}(G) = 2 = \left\lceil \frac{n}{1+\Delta_{sb}} \right\rceil$. Thus the bound is sharp. □

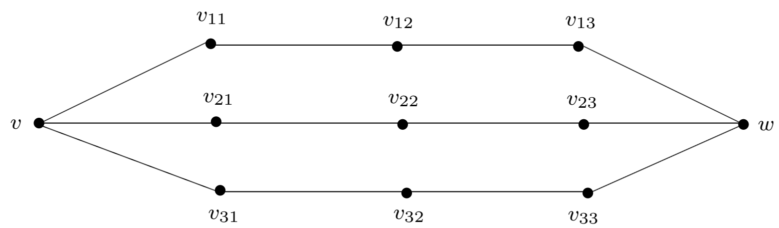

Figure 3. **A graph G with $\gamma_{sb}(G) = \left\lceil \frac{n}{1+\Delta_{sb}} \right\rceil$.**

Theorem 13. *Let $\Pi = (\text{ds}(v_1), \text{ds}(v_2), \ldots, \text{ds}(v_n))$ be the* ds*-sequence of a graph and let $\text{ds}(v_1) \geq \text{ds}(v_2) \geq \cdots \geq \text{ds}(v_n)$. Let $t = \min\{k : k + \text{ds}(v_1) + \text{ds}(v_2) + \cdots + \text{ds}(v_k) \geq n\}$. Then $\gamma_{sb}(G) \geq t$ and the bound is sharp.*

Proof. Let S be any subset of V with $|S| = r < t$. Hence $r + \text{ds}(v_1) + \text{ds}(v_2) + \cdots + \text{ds}(v_t) < n$. Also $\sum_{v\in S} \text{ds}(v) \leq \text{ds}(v_1) + \text{ds}(v_2) + \cdots + \text{ds}(v_t)$. Hence $|S| + \sum_{v\in S} \text{ds}(v) < n$. Thus

$$|S| + \sum_{v\in S} \left(\sum_{u\neq v} \frac{1}{d(v,u)} \right) < n. \tag{3.4}$$

Now, suppose S is a sb-dominating of G.
Then $\sum_{v\in S} \frac{1}{d(u,v)} \geq 1$ for all $u \in V - S$. Hence $\sum_{u\in V-S} \left(\sum_{v\in S} \frac{1}{d(u,v)} \right) \geq n - |S|$. Therefore $|S| + \sum_{u\in V-S} \left(\sum_{v\in S} \frac{1}{d(u,v)} \right) \geq n$ which contradicts (4). Hence S is not a sb-dominating set of G. So $\gamma_{sb}(G) \geq t$. □

4. *sb*-domination and diameter

In this section we present several basic results and bounds for γ_{sb} in terms of the diameter of a graph.

Observation 14. If diam$(G)=1$, then $G=K_n$ and $\gamma_{sb}(G)=diam(G)=1$. If diam$(G)=2$, then $\gamma_{sb}(G)=\begin{cases}1 & \text{if } \Delta=n-1\\ 2 & \text{otherwise.}\end{cases}$

Lemma 1. *Let G be a connected graph. Then $\gamma_{sb}(G)\le diam(G)$ and the bound is sharp.*

Proof. Let diam$(G)=d$ and let $D=\{v_1,v_2,\ldots,v_d\}$ be any subset of V with $|D|=diam(G)=d$. Since $d(u,v)\le d$ for all $u,v\in V$, it follows that D is a *sb*-dominating set of G and hence $\gamma_{sb}(G)\le d$. By Observation 14, it follows that equality holds if $d=1$ or $d=2$ and $\Delta\ne n-1$. □

For any positive integer t, there exists a graph G with $\gamma_{sb}(G)=diam(G)=t$, as shown in the following theorem.

Theorem 15. *For any positive integer t, there exists a graph G with $\gamma_{sb}(G)=diam(G)=t$.*

Proof. Let $G=K_t\Box K_t\Box\cdots\Box K_t$ be the Cartesian product of t copies of K_t. Clearly, diam$(G)=t$ and hence $\gamma_{sb}(G)\le t$. Now let $D=\{v_1,v_2,\ldots,v_{t-1}\}$ be any subset of $V(G)$. Let $V_i=\{u_{i_1},u_{i_2},\ldots,u_{i_t}\}$ be the vertex set of the i^{th} copy of K_t in G. Let $v_j=(u_{1j_1},u_{2j_2},\ldots,u_{tj_t})$, where $1\le j\le t-1$. Let $1\le r\le t$. Since $|D|=t-1$, we can choose $u_{rk_r}\in V_r$ such that $u_{rk_r}\ne u_{rj_r}$ for all j with $1\le j\le t-1$. Let $u=(u_{1k_1},u_{2k_2},\ldots,u_{tk_t})$. Since u differs from v_j in all the t coordinates $d(u,v_j)=t$. Hence $ds(D,u)=\frac{t-1}{t}<t$. Thus D is not a *sb*-dominating set of G and hence $\gamma_{sb}(G)\ge t$. Thus $\gamma_{sb}(G)=t=$diam(G). □

Observation 16. Let r be the radius of G and let $Z(G)$ denote the centre of G. If $|Z(G)|\ge r$, then $Z(G)$ is a *sb*-dominating set of G and hence $\gamma_{sb}(G)\le r$.

Theorem 17. *Let G be a connected graph of order n with $\gamma_{sb}(G)=2$. Then diam$(G)\le 6$.*

Proof. Let $D=\{u,v\}$ be a *sb*-dominating set of G. If D is a dominating set of G, then it follows from Theorem 4 that diam$(G)\le 5$. Suppose D is not a dominating set. Let $D_1=N[u]\cup N[v]$ and $D_2=V-D_1$. Clearly $D_2\ne\emptyset$. Since D is a *sb*-dominating set of G, it follows that $d(x,u)=d(x,v)=2$ for all $x\in D_2$. Hence $d(x,y)\le 4$ if $x,y\in D_2$ and $d(x,y)\le 3$ if $x\in D_2$ and $y\in D_1$. Now, let $x,y\in D_1$. If $x,y\in N(u)$ or $x,y\in N(v)$ then $d(x,y)\le 2$. If $x\in N(u)$ and $y\in N(u)$, let $w\in D_2$. Then $d(x,y)\le d(x,u)+d(u,w)+d(w,v)+d(v,y)=6$. Thus $d(x,y)\le 6$ for all $x,y\in V$ and hence diam$(G)\le 6$. □

Theorem 18. *Let k be a positive integer with $2\le k\le 6$. Then there exists a graph G such that $\gamma_{sb}(G)=2,\gamma(G)>2$ and diam$(G)=k$.*

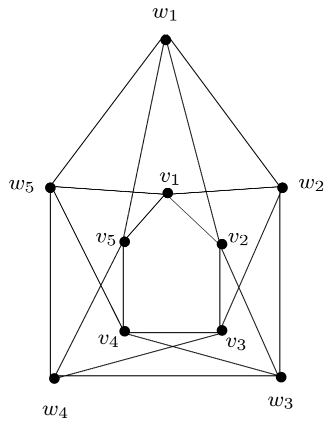

Figure 4. **A graph G with $\gamma_{sb}(G) = diam(G) = 2$ and $\gamma(G) > 2$.**

Proof. If $k = 5$ then for the graph G given in Figure 1, $\gamma_{sb}(G) = 2$, $\gamma(G) = 3$ and $\text{diam}(G) = 5$. If $k = 6$, then for the path $G = P_7 = (v_1, v_2, v_3, v_4, v_5, v_6, v_7)$, we have $\gamma_{sb}(G) = 2, \gamma(G) = 3$ and $\text{diam}(G) = 6$. If $k = 4$, then let $G = P_3 \circ K_1$ where $P_3 = v_1v_2v_3$ and let w_i be the pendent vertex adjacent to v_i. Then $\{v_1, v_3\}$ is a γ_{sb}-set of $G, \{v_1, v_2, v_3\}$ is a γ-set of G and $d(w_1, w_3) = 4$. Thus $\gamma_{sb}(G) = 2, \gamma(G) = 3$ and $\text{diam}(G) = 4$.

Assume now that $k = 2$. For the graph G given in Figure 4, $\text{diam}(G) = 2$ and $\Delta(G) \leq n - 2$. Hence $\gamma_{sb}(G) = 2$. Also for any two vertices x, y in G, $N(x) \cap N(y) \neq \emptyset, |N(x)| = |N(y)| = 4$ and hence $|N[x] \cup N[y]| \leq 9$. Thus $\{x, y\}$ is not a dominating set of G and hence $\gamma(G) > 2$.

Finally let $k = 3$. Let G be the graph obtained from $K_4 \circ K_1$ by removing one pendent vertex. Let $V(K_4) = \{v_1, v_2, v_3, v_4\}$ and let w_i be the pendent vertex adjacent to $v_i, 1 \leq i \leq 3$. Then $D = \{v_1, v_2\}$ is a γ_{sb}-set of G, $D_1 = \{v_1, v_2, v_3\}$ is a γ-set of G and $d(w_1, w_3) = 3$. Thus $\gamma_{sb}(G) = 2, \gamma(G) = 3$ and $\text{diam}(G) = 3$. □

5. Conclusion and Scope

The dominating strength ds(v) of a vertex is a topological index which is useful in identifying the most influential member in a social network. Also the concept of sb-domination is a natural generalization of domination and disjunctive domination in graphs. A study of total sb-domination, independent sb-domination and algorithmic aspects of sb-domination are a few promising directions for further research.

Conflict of Interest: The authors declare that they have no conflict of interest.

Data Availability: Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

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