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Research Article

Edge corona product and its topological descriptors with applications in complex molecular structures

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Abstract: Graph operations offer a robust framework that enables the analysis, modeling, and resolution of intricate problems. Their versatility and broad range of applications make them essential across numerous fields of study and research, playing an irreplaceable role in tackling complex challenges. A topological index is a real number associated with a graph that gives insight into the topological properties of the graph. There are numerous topological indices in this era now, with three variants like degree based, distance based and eccentricity based topological indices. In this paper, we studied a well known graph operation named as edge corona product and investigate their some degree based topological indices. As applications, this graph operations can be used to study topological properties of complex structure of linear and cyclic silicate networks, together with triangular and double triangular networks. Some existing results in the literature can be obtained as corollaries of the new results. A conjecture is proposed relating the general first Zagreb index of the edge corona product of two graphs.

Keywords: Graph operation, topological index, complex molecular structures.

AMS Subject classification: 05C10, 05C30, 05C90, 05C92

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1. Introduction

Graph theory is the study of graphs that deals with the relationship between edges and vertices. It has wide applications in the filed of computer science, information technology, electrical engineering, bio-sciences, linguistics, chemistry and mathematics [28]. In the field of chemistry, sometimes it is too hard and difficult to study the complex and huge chemical structures. Therefore, chemical structures are studied by converting atoms to vertices and covalent bonds to the edges [9]. As a result, molecule is transformed into a molecular graph. Most of the definitions in this research paper are taken from [7, 8, 14, 16].

Let $\zeta(V, E)$ be a simple and undirected graph, where $V(\zeta)$ and $E(\zeta)$ are the vertex set and edge set, respectively. The degree of a vertex v is defined as the number of incident edges of the graph ζ to v. It is denoted by $d(v)$. The degree of an edge $e = uv \in E(\zeta)$ of a graph ζ is $d(e) = d(u) + d(v) - 2$. The order (number of vertices) and size (number of edges) of a graph ζ will be denoted by n and m, respectively. A number that is invariant under isomorphism is referred to as a topological graph index. This value is also known as a graph theoretical descriptor or a molecular structure descriptor [20]. The most prime applications of the topological indices are as follows: They are useful to predict the physio-chemical properties of molecular structures, and they are helpful for the study of QSAR/QSPR analysis.

Gutman and Trinajstić [18], presented the first and second Zagreb indices as

$$M_1(\zeta) = \sum_{v \in V(\zeta)} d(v)^2 = \sum_{uv \in E(\zeta)} [d(u) + d(v)] \text{ and } M_2(\zeta) = \sum_{uv \in E(\zeta)} d(u)d(v).$$

These topological indices were initially used to determine the total π-electron energy of molecular graphs in 1972. Later, the Zagreb indices picked up significance for QSPR/QSAR modeling.

Miličević et al. [26], in 2004 reformulated the Zagreb index in terms of edge degrees with

$$EM_1(\zeta) = \sum_{e \in E(\zeta)} d(e)^2 = \sum_{uv \in E(\zeta)} (d(u) + d(v) - 2)^2.$$

The reduced first and second Zagreb indices are defined by

$$RM_1(\zeta) = \sum_{v \in V(\zeta)} (d(v) - 1)^2 \quad \text{and} \quad RM_2(\zeta) = \sum_{uv \in E(\zeta)} (d(u) - 1)(d(v) - 1).$$

In 2013, Shirdel [29], proposed a new version of Zagreb indices called first hyper-Zagreb index which is

$$HM_1(\zeta) = \sum_{uv \in E(\zeta)} (d(u) + d(v))^2.$$

Furtula and Gutman [17], studied the forgotten topological index

$$F(\zeta) = \sum_{v \in V(\zeta)} d(v)^3.$$

Alameri *et al.* [3], studied the Yemen index (y-index) as

$$Y(\zeta) = \sum_{v \in V(\zeta)} d(v)^4.$$

Graph operations sometimes called as graph products play a significant role in computer science, network theory as well as in pure and applied mathematics. For instance, a key model for connecting computers is the cartesian product. It is vital to look for Hamiltonian pathways and cycles in the network in order to synchronise the operation of the entire framework [31].

In [24], authors evaluated important results on some graph operations for first and second Zagreb indices. Khaksari *et al.* [22], found the exact formulae for the forgotten index for some graph operations. Alameri *et al.* [3], investigated the y-index for some graph operations. Also computed the same index for some classes of nano-structures like nano-tube and nano-torus. Akhter *et al.* [2], calculated the Mostar index for some graph products. The hyper-Wiener index, multiplicative Zagreb index and Zagreb co-indices for some operations studied in [23], [10] and [6], respectively. Arezoomand and Taeri [5], presented the exact expressions of Zagreb indices for the generalized hierarchical products. Nilanjan De [11], computed the forgotten topological index of different corona products of graphs. See e.g. some related articles [15, 21, 29, 30].

Following is the definition of the new graph operation introduced in this paper.

Definition 1. (edge corona product) For two graphs $\zeta_1(V(\zeta_1),E(\zeta_1))$ and $\zeta_2(V(\zeta_2),E(\zeta_2))$, the edge corona product of ζ_1 and ζ_2 denoted by $\zeta_1 \triangle \zeta_2$ is defined as the graph obtained by adding a copy of ζ_2 for each edge $uv \in E(\zeta_1)$ and u and v are adjacent with every vertex of that copy of ζ_2.

An example of edge corona product of two graphs is presented in Figure 1.
In this paper, we investigated the exact formulae for some degree based topological indices for this operation. Our main results generated the formulae for some degree based topological indices of chain and cyclic silicates very easily as compared to the techniques presented in [4, 19]. A conjecture to establish the formula of the general first Zagreb index for the edge corona product of graphs for any integer $\alpha \geq 2$ is given.

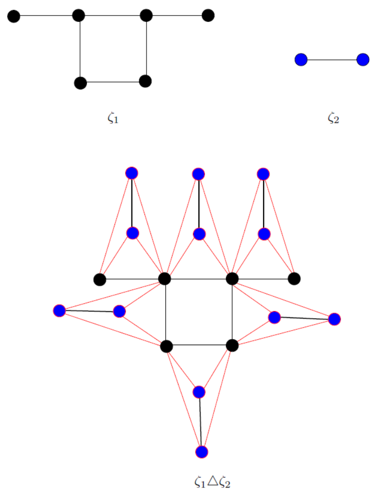

Figure 1. **Illustration of edge corona product of two graphs.**

2. Results

2.1. Fundamental properties

In this section, we first discuss the order, size and degree behavior of the edge corona product. Further, the exact expressions for some degree based topological invariants are investigated. After each theorem about some index, we establish expression of the same index for the n-dimensional chain and cyclic silicate structures as corollaries. In the following, $C(\zeta_2)$ denotes one of the copies of the graph ζ_2.

Lemma 1. *Let $\zeta_1(n_1, m_1)$ and $\zeta_2(n_2, m_2)$ be two graphs. Then the order and the size of $\zeta_1 \triangle \zeta_2(n_3, m_3)$ are given by*

i. $n_3 = m_1 n_2 + n_1$

ii. $m_3 = m_1(2n_2 + m_2 + 1)$.

The following lemma gives the information about the degrees of each vertex in the edge corona product:

Lemma 2. *Let $\zeta_1(n_1, m_1)$ and $\zeta_2(n_2, m_2)$ be two graphs. Then for each $v \in V(\zeta_1 \triangle \zeta_2(n_3, m_3))$, we have*

$$d_{\zeta_1 \triangle \zeta_2}(v) = \begin{cases} d_{\zeta_1}(v)(1 + n_2); & if v \in V(\zeta_1) \\ d_{\zeta_2}(v) + 2; & if v \in V(C(\zeta_2)). \end{cases}$$

Having some idea about the degrees of vertices, we can give the degree sequence of the edge corona product of two graphs:

Corollary 1. *Let the degree sequences of the graphs ζ_1 and ζ_2 be $D_1 = \{d_1^{(a_1)},\ d_2^{(a_2)},\ \cdots, d_k^{(a_k)}\}$ and $D_2 = \{e_1^{(b_1)},\ e_2^{(b_2)},\ \cdots, e_k^{(b_k)}\}$, respectively. Then the degree sequence of $\zeta_1 \triangle \zeta_2$ is*

$$D(\zeta_1\triangle\zeta_2) = \Big\{(d_1(1+n_2))^{(a_1)},\ (d_2(1+n_2))^{(a_2)},\ \cdots, (d_k(1+n_2))^{(a_k)},$$

$$(e_1+2)^{(b_1 m_1)}, (e_2+2)^{(b_2 m_1)},\ \cdots,\ (e_t+2)^{(b_t m_1)}\Big\}.$$

In [12], a new graph invariant carrying a lot of combinatoric and graph theoretical properties is defined and its properties are primarily investigated in [12, 13]. This invariant is related to Euler invariant and also the cyclomatic number:

Definition 2. [12] Let $D = \{1^{(a_1)}, 2^{(a_2)}, 3^{(a_3)}, \cdots, \Delta^{(a_\Delta)}\}$ be a set which also is the degree sequence of a graph ζ. The $\Omega(\zeta)$ of the graph ζ is defined in terms of the degree sequence as

$$\begin{aligned}\Omega(\zeta) = &\ a_3 + 2a_4 + 3a_5 + \cdots + (\Delta-2)a_\Delta - a_1 \\ = &\sum_{i=1}^{\Delta}(i-2)a_i.\end{aligned}$$

In the following result, we calculate omega invariant of the edge corona products:

Theorem 1. *The omega invariant of $\zeta_1\triangle\zeta_2$ is*

$$\Omega(\zeta_1\triangle\zeta_2) = 2[m_1(m_2+n_2+1) - n_1].$$

The number of faces of the edge corona product can easily be deduced by Corollary 3.1 in [12].

Theorem 2. *The number of faces of the edge corona product $\zeta_1\triangle\zeta_2$ is*

$$r(\zeta_1\triangle\zeta_2) = 1 + m_1(m_2+n_2+1) - n_1.$$

Proof. By Corollary 3.1 in [12] and by the fact that the edge corona product is connected, the result follows. □

2.2. Topological graph indices of edge corona products

The first main result is about the edge corona product and the first Zagreb index.

Theorem 3. *[27] Let $\zeta_1(n_1, m_1)$ and $\zeta_2(n_2, m_2)$ be two graphs. Then the first Zagreb index of $\zeta_1\triangle\zeta_2$ is*

$$M_1(\zeta_1\triangle\zeta_2) = (1+n_2)^2 M_1(\zeta_1) + m_1[M_1(\zeta_2) + 8m_2 + 4n_2].$$

Theorem 4. *[27] Let $\zeta_1(n_1, m_1)$ and $\zeta_2(n_2, m_2)$ be two graphs. The formula of the second Zagreb index of $\zeta_1 \triangle \zeta_2$ is*

$$M_2(\zeta_1 \triangle \zeta_2) = (1+n_2)^2 M_2(\zeta_1) + m_1\Big[M_2(\zeta_2) + 2M_1(\zeta_2) + 4m_2\Big] + 2M_1(\zeta_1)(1+n_2)(n_2+m_2).$$

Theorem 5. *For $\zeta_1(n_1, m_1)$ and $\zeta_2(n_2, m_2)$ graphs, the first reformulated Zagreb index of the edge corona product $\zeta_1 \triangle \zeta_2$ is equal to*

$$EM_1(\zeta_1 \triangle \zeta_2) = (1+n_2)^2 HM_1(\zeta_1) + m_1 HM_1(\zeta_2) + 4M_1(\zeta_1)\Big[m_2 + m_2 n_2 - 1 - n_2\Big] + 6m_1 M_1(\zeta_2) + n_2(1+n_2)^2 F(\zeta_1) + 4m_1(m_2+1).$$

Proof. Applying the definition of the first reformulated Zagreb index on the edge corona product of graphs and then using Lemma 2, we get

$$\begin{aligned}
EM_1(\zeta_1 \triangle \zeta_2) &= \sum_{uv \in E(\zeta_1 \triangle \zeta_2)} [\ d_{\zeta_1 \triangle \zeta_2}(e)\ ]^2 \\
&= \sum_{uv \in E(\zeta_1 \triangle \zeta_2)} [\ d_{\zeta_1 \triangle \zeta_2}(u) + d_{\zeta_1 \triangle \zeta_2}(v) - 2\ ]^2 \\
&= \sum_{uv \in E(\zeta_1)} [\ d_{\zeta_1 \triangle \zeta_2}(u) + d_{\zeta_1 \triangle \zeta_2}(v) - 2\ ]^2 + \sum_{uv \in E(\zeta_1)} \sum_{uv \in E(\zeta_2),} [\ d_{\zeta_1 \triangle \zeta_2}(u) \\
&+ d_{\zeta_1 \triangle \zeta_2}(v) - 2\ ]^2 + \sum_{\substack{uv \in E(\zeta_1 \triangle \zeta_2) \\ u \in V(\zeta_1),\ v \in V(\zeta_2)}} [\ d_{\zeta_1 \triangle \zeta_2}(u) + d_{\zeta_1 \triangle \zeta_2}(v) - 2\ ]^2 \\
&= \sum_{uv \in E(\zeta_1)} \Big[\ (1+n_2) d_{\zeta_1}(u) + (1+n_2) d_{\zeta_1}(v) - 2\ \Big]^2 + m_1 \sum_{uv \in E(\zeta_2)} \Big[d_{\zeta_2}(u) + 2 \\
&+ d_{\zeta_2}(v) + 2 - 2\Big]^2 + \sum_{\substack{uv \in E(\zeta_1 \triangle \zeta_2) \\ u \in V(\zeta_1),\ v \in V(\zeta_2)}} \Big[(1+n_2) d_{\zeta_1}(u) + d_{\zeta_2}(v) + 2 - 2\Big]^2 \\
&= (1+n_2)^2 HM_1(\zeta_1) + m_1 HM_1(\zeta_2) + 4M_1(\zeta_1)\Big[m_2 + m_2 n_2 - 1 - n_2\Big] \\
&+ 6m_1 M_1(\zeta_2) + n_2(1+n_2)^2 F(\zeta_1) + 4m_1(m_2+1).
\end{aligned}$$

□

Theorem 6. *Let $\zeta_1(n_1, m_1)$ and $\zeta_2(n_2, m_2)$ be two graphs. Then the reduced first Zagreb index of the $\zeta_1 \triangle \zeta_2$ is given by the formula*

$$RM_1(\zeta_1 \triangle \zeta_2) = (1+n_2)^2 M_1(\zeta_1) + n_1 - 4m_1(1+n_2) + m_1[M_1(\zeta_2) + n_2 + 4m_2].$$

Proof. Apply the definition of reduced first Zagreb index and depending on the types of edges of the $\zeta_1 \triangle \zeta_2$ and Lemma 2, we obtained

$$\begin{aligned}
RM_1(\zeta_1 \triangle \zeta_2) &= \sum_{v \in V(\zeta_1 \triangle \zeta_2)} \Big[d_{\zeta_1 \triangle \zeta_2}(v) - 1\Big]^2 \\
&= \sum_{v \in V(\zeta_1)} \Big[d_{\zeta_1 \triangle \zeta_2}(v) - 1\Big]^2 + \sum_{uv \in E(\zeta_1)} . \sum_{v \in V(\zeta_2)} \Big[d_{\zeta_1 \triangle \zeta_2}(v) - 1\Big]^2 \\
&= \sum_{v \in V(\zeta_1)} \Big[d_{\zeta_1}(v)(1+n_2) - 1\Big]^2 + m_1 \sum_{v \in V(\zeta_2)} \Big[d_{\zeta_2}(v) + 2 - 1\Big]^2 \\
&= \sum_{v \in V(\zeta_1)} \Big[d_{\zeta_1}(v)(1+n_2) - 1\Big]^2 + m_1 \sum_{v \in V(\zeta_2)} \Big[d_{\zeta_2}(v) + 1\Big]^2 \\
&= \sum_{v \in V(\zeta_1)} \Big[d_{\zeta_1}(v)^2.(1+n_2)^2 + 1 - 2(1+n_2)d_{\zeta_1}(v)\Big] + m_1 \sum_{v \in V(\zeta_2)} \Big[d_{\zeta_2}(v)^2 + 2d_{\zeta_2}(v) + 1\Big] \\
&= (1+n_2)^2 M_1(\zeta_1) + n_1 - 2(1+n_2)(2m_1) + m_1\Big[M_1(\zeta_2) + 2(2m_2) + n_2\Big] \\
&= (1+n_2)^2 M_1(\zeta_1) + n_1 - 4m_1(1+n_2) + m_1\Big[M_1(\zeta_2) + n_2 + 4m_2\Big].
\end{aligned}$$

□

Theorem 7. *[1] The exact expression for the first hyper-Zagreb index for the edge corona product of $\zeta_1(n_1, m_1)$ and $\zeta_2(n_2, m_2)$ is*

$$\begin{aligned}
HM_1(\zeta_1 \triangle \zeta_2) = (1+n_2)^2 HM_1(\zeta_1) &+ m_1\Big[HM_1(\zeta_2) + 16m_2 + 8M_1(\zeta_2)\Big] \\
&+ n_2(1+n_2)^2 F(\zeta_1) + 2m_1 M_1(\zeta_2) \\
&+ 8m_1 n_2 + 4(1+n_2)(n_2+m_2)M_1(\zeta_1) + 16m_1 m_2.
\end{aligned}$$

Theorem 8. *The forgotten index of $\zeta_1 \triangle \zeta_2$ is equal to*

$$F(\zeta_1 \triangle \zeta_2) = (1+n_2)^3 F(\zeta_1) + m_1\Big[F(\zeta_2) + 6M_1(\zeta_2) + 24m_2 + 8n_2\Big].$$

Proof. By using definition of the forgotten index and the Lemma 2, we have

$$\begin{aligned}
F(\zeta_1 \triangle \zeta_2) &= \sum_{v \in V(\zeta_1 \triangle \zeta_2)} d_{\zeta_1 \triangle \zeta_2}(v)^3 \\
&= \sum_{v \in V(\zeta_1)} d_{\zeta_1 \triangle \zeta_2}(v)^3 + \sum_{uv \in E(\zeta_1)} \sum_{v \in V(\zeta_2)} d_{\zeta_1 \triangle \zeta_2}(v)^3 \\
&= \sum_{v \in V(\zeta_1)} \Big[d_{\zeta_1}(v)(1+n_2)\Big]^3 + m_1 \sum_{v \in V(\zeta_2)} \Big[d_{\zeta_2}(v) + 2\Big]^3
\end{aligned}$$

$$= \sum_{v\in V(\zeta_1)} d_{\zeta_1}(v)^3(1+n_2)^3 + m_1 \sum_{v\in V(\zeta_2)} \Big[d_{\zeta_2}(v)^3 + (2)^3 + 3.d_{\zeta_2}(v)^2.2 + 3.d_{\zeta_2}(v).(2)^2 \Big]$$

$$= (1+n_2)^3 \sum_{v\in V(\zeta_1)} d_{\zeta_1}(v)^3 + m_1 \sum_{v\in V(\zeta_2)} \Big[d_{\zeta_2}(v)^3 + 8 + 6d_{\zeta_2}(v)^2 + 12d_{\zeta_2}(v) \Big]$$

$$= (1+n_2)^3 F(\zeta_1) + m_1 \Big[\sum_{v\in V(\zeta_2)} d_{\zeta_2}(v)^3 + 8 \sum_{v\in V(\zeta_2)} + 6 \sum_{v\in V(\zeta_2)} d_{\zeta_2}(v)^2 + 12 \sum_{v\in V(\zeta_2)} d_{\zeta_2}(v) \Big]$$

$$= (1+n_2)^3 F(\zeta_1) + m_1 \Big[F(\zeta_2) + 6M_1(\zeta_2) + 24m_2 + 8n_2 \Big].$$

□

Theorem 9. *Let $\zeta_1(n_1, m_1)$ and $\zeta_2(n_2, m_2)$ be two graphs and $\zeta_1 \triangle \zeta_2$ be their edge corona product, then the reduced second Zagreb index is given as*

$$RM_2(\zeta_1 \triangle \zeta_2) = (1+n_2)^2 M_2(\zeta_1) + \Big(2m_2 + 2m_2n_2 + (n_2)^2 - 1\Big) M_1(\zeta_1)$$
$$+ m_1 \Big[M_1(\zeta_2) + M_2(\zeta_2) - 3m_2 - 2n_2 \Big].$$

Proof. Applying the definition of the reduced second Zagreb index and the Lemma 2, we obtained

$$RM_2(\zeta_1 \triangle \zeta_2) = \sum_{uv\in E(\zeta_1 \triangle \zeta_2)} \Big[d_{\zeta_1 \triangle \zeta_2}(u) - 1 \Big] \Big[d_{\zeta_1 \triangle \zeta_2}(v) - 1 \Big]$$

$$= \sum_{uv\in E(\zeta_1)} \Big[d_{\zeta_1}(u) + n_2 d_{\zeta_1}(u) - 1 \Big] \Big[d_{\zeta_1}(v) + n_2 d_{\zeta_1}(v) - 1 \Big] + m_1 \sum_{uv\in E(\zeta_2)} \Big[d_{\zeta_2}(u) + 2 - 1 \Big].$$

$$\Big[d_{\zeta_2}(v) + 2 - 1 \Big] + \sum_{\substack{uv\in E(\zeta_1 \triangle \zeta_2) \\ u\in V(\zeta_1),\ v\in V(\zeta_2)}} \Big[d_{\zeta_1}(u) + n_2 d_{\zeta_1}(u) - 1 \Big] \Big[d_{\zeta_2}(v) + 2 - 1 \Big]$$

$$= \sum_{uv\in E(\zeta_1)} \Big[(1+n_2)^2 d_{\zeta_1}(u) d_{\zeta_1}(v) - n_2 \Big(d_{\zeta_1}(u) + d_{\zeta_1}(v) \Big) - \Big(d_{\zeta_1}(u) + d_{\zeta_1}(v) \Big) \Big]$$

$$+ m_1 \sum_{uv\in E(\zeta_2)} \Big[d_{\zeta_2}(u) d_{\zeta_2}(v) + \Big(d_{\zeta_2}(u) + d_{\zeta_2}(v) \Big) + 1 \Big] + \sum_{\substack{uv\in E(\zeta_1 \triangle \zeta_2) \\ u\in V(\zeta_1),\ v\in V(\zeta_2)}} \Big[(1+n_2) d_{\zeta_1}(u) d_{\zeta_2}(v)$$

$$+ (1+n_2) d_{\zeta_1}(u) - d_{\zeta_2}(v) - 1 \Big]$$

$$= (1+n_2)^2 M_2(\zeta_1) - n_2 M_1(\zeta_1) - M_1(\zeta_1) + m_1 M_2(\zeta_2) + m_1 M_1(\zeta_2) + m_1 m_2$$
$$+ (1+n_2).2m_2 M_1(\zeta_1) + (1+n_2).n_2 M_1(\zeta_1) - 4m_1m_2 - 2n_2m_1$$
$$= (1+n_2)^2 M_2(\zeta_1) + \Big(2m_2 + 2m_2n_2 + (n_2)^2 - 1\Big) M_1(\zeta_1)$$
$$+ m_1 \Big[M_1(\zeta_2) + M_2(\zeta_2) - 3m_2 - 2n_2 \Big].$$

□

Theorem 10. *Let $\zeta_1(n_1, m_1)$ and $\zeta_2(n_2, m_2)$ be two graphs, then the formula of Yemen index for the edge corona product of the graph $\zeta_1 \triangle \zeta_2$ is*

$$Y(\zeta_1 \triangle \zeta_2) = (1+n_2)^4 Y(\zeta_1) + m_1\Big[Y(\zeta_2) + 16n_2 + 8F(\zeta_2) + 64m_2 + 24M_1(\zeta_2)\Big].$$

Proof. By using definition of Yemen index on the $\zeta_1 \triangle \zeta_2$, we obtained

$$Y(\zeta_1 \triangle \zeta_2) = \sum_{v \in V(\zeta_1 \triangle \zeta_2)} d_{\zeta_1 \triangle \zeta_2}(v)^4.$$

Using Lemma 2

$$\begin{aligned}
Y(\zeta_1 \triangle \zeta_2) &= \sum_{v \in V(\zeta_1)} d_{\zeta_1 \triangle \zeta_2}(v)^4 + \sum_{uv \in E(\zeta_1)} \sum_{v \in V(\zeta_2)} d_{\zeta_1 \triangle \zeta_2}(v)^4 \\
&= \sum_{v \in V(\zeta_1)} \Big[d_{\zeta_1}(v).(1+n_2)\Big]^4 + m_1 . \sum_{v \in V(\zeta_2)} \Big[d_{\zeta_2}(v) + 2\Big]^4 \\
&= \sum_{v \in V(\zeta_1)} d_{\zeta_1}(v)^4 (1+n_2)^4 + m_1 \sum_{v \in V(\zeta_2)} \Big[d_{\zeta_2}(v)^4 + (2)^4 + 4d_{\zeta_2}(v)^3.2 + 4.d_{\zeta_2}(v).(2)^3 \\
&+ 6.d_{\zeta_2}(v)^2.(2)^2\Big] \\
&= (1+n_2)^4 \sum_{v \in V(\zeta_1)} d_{\zeta_1}(v)^4 + m_1\Big[\sum_{v \in V(\zeta_2)} d_{\zeta_2}(v)^4 + 16 \sum_{v \in V(\zeta_2)} 1 + 8 \sum_{v \in V(\zeta_2)} d_{\zeta_2}(v)^3 + 32 \sum_{v \in V(\zeta_2)} d_{\zeta_2}(v) \\
&+ 24 \sum_{v \in V(\zeta_2)} d_{\zeta_2}(v)^2\Big] \\
&= (1+n_2)^4 Y(\zeta_1) + m_1\Big[\ Y(\zeta_2) + 16n_2 + 8F(\zeta_2) + 64m_2 + 24M_1(\zeta_2)\ \Big].
\end{aligned}$$

□

3. Applications of the edge corona product of graphs

Silicates are the minerals that contains silicon and oxygen in tetrahedral SiO_4^{4-} units, all these units linked together in several patterns form like chain silicates and cyclic silicates. Chain silicates: these units contain $(SiO_3)_n^{2n-}$ ions; that formed by linking three or more than three or n number of tetrahedral SiO_4^{4-} units cyclically. Each of them shares two oxygen atoms with other units of the structure. Cyclic silicates: they are also named as pyroxenes, that contain $(SiO_3)_n^{2n-}$ ions, that formed by linking n number of tetrahedral SiO_4^{4-} units linearly. Each unit shares two oxygen atoms with other units. N-dimensional network: from cyclic silicates and chain silicate, we have define their n dimensional structures by linking their n units. Authors are trying to find the degree base topological indices degrees of the n dimensional chain silicate network as well as n dimensional cyclic silicate network, [19, 25].

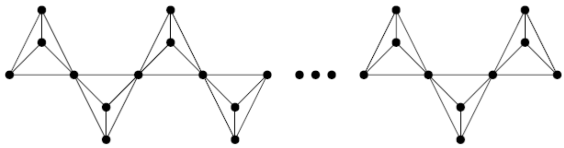

Figure 2. **Construction of n-dimensional linear silicate chain**

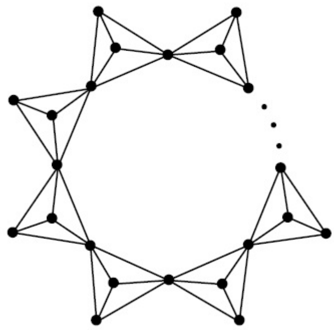

Figure 3. **Construction of n-dimensional cyclic silicate chain**

It is of great interest and important task to develop new graph operations that can construct complex networks. The novel edge corona product of graphs has the ability to construct various graphs networks. Some of its applications are discussed below. By considering the definition of the edge corona product of graphs we can construct the complex networks of linear and chain silicates and triangular and double triangular snake graphs as follows:

i. If we consider $\zeta_1 = P_k$ and $\zeta_2 = P_2$, then by applying edge corona product on these graphs we can obtain linear silicate network, as shown in Figure 2.

ii. Taking $\zeta_1 = C_k$ and $\zeta_2 = P_2$ we can generate cyclic silicate network, this is illustrated in Figure 3.

iii. The edge corona product of P_k and K_1 (an isolated vertex) generates triangular

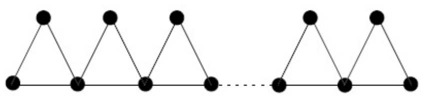

Figure 4. **Construction of n-dimensional triangular chain**

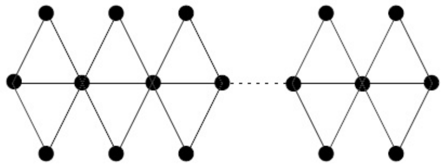

Figure 5. **Construction of n-dimensional double triangular chain**

snake graph, this is illustrated in Figure 4.

iv. Double triangular snake graph can be generated by taking the edge corona product of P_k and $\overline{K_2}$ (two isolated vertices), this is shown in Figure 5.

Some degree based topological indices of the silicate chains and triangular snake graphs are investigated in [4, 19] by long computation. By applying our main results, these results can be produced easily. Tables 1-4 represent the formulas of the linear and cyclic silicate networks, and the triangular and double triangular networks of some degree based topological indices using Theorems 3-10.

Table 1. **Topological indices of linear silicate (LSC) chain as applications of our results.**

For LSC replace $\zeta_1 = P_k$ and $\zeta_2 = P_2$ in	Topological index	Formula
Theorem 3	First Zagreb index	54k-72
Theorem 4	Second Zagreb index	117k-189
Theorem 5	First reformulated Zagreb index	312k-546
Theorem 6	Reduced first Zagreb index	33k-50
Theorem 7	Hyper-Zagreb index	504k-810
Theorem 8	Forgotten index	270k-432
Thoerem 9	Reduced second Zagreb index	68k-122
Theorem 10	Yemen index	1458k-2592

Table 2. **Topological indices of cyclic silicate chain (CSC) as applications of our results.**

For CSC replace $\zeta_1 = C_k$ and $\zeta_2 = P_2$ in	Topological index	Formula
Theorem 3	First Zagreb index	54k
Theorem 4	Second Zagreb index	117k
Theorem 5	First reformulated Zagreb index	312k
Theorem 6	Reduced first Zagreb index	33k
Theorem 7	Hyper Zagreb index	504k
Theorem 8	Forgotten index	270k
Thoerem 9	Reduced second Zagreb index	68k
Theorem 10	Yemen index	1458k

Table 3. Topological indices of triangular snake graph (TSG) as applications of our results.

For TSG replace $\zeta_1 = P_k$ and $\zeta_2 = K_1$ in	Topological index	Formula
Theorem 3	First Zagreb index	20k-28
Theorem 4	Second Zagreb index	32k-56
Theorem 5	First reformulated Zagreb index	68k-132
Theorem 6	Reduced first Zagreb index	10k-17
Theorem 7	Hyper Zagreb index	136k-232
Theorem 8	Forgotten index	72k-120
Thoerem 9	Reduced second Zagreb index	14k-30
Theorem 10	Yemen index	272k-496

Table 4. Topological indices of double triangular snake graph (DTSG) as applications of our results.

For DTSG replace $\zeta_1 = P_k$ and $\zeta_2 = \overline{K_2}$ in	Topological index	Formula
Theorem 3	First Zagreb index	44k-62
Theorem 4	Second Zagreb index	84k-144
Theorem 5	First reformulated Zagreb index	244k-454
Theorem 6	Reduced first Zagreb index	27k-44
Theorem 7	Hyper Zagreb index	400k-682
Theorem 8	Forgotten index	232k-394
Thoerem 9	Reduced second Zagreb index	44k-86
Theorem 10	Yemen index	1328k-2462

Considering the results of the above theorems 3, 8 and 10, we get the results below:

$$\begin{aligned}
M_1(\zeta_1 \triangle \zeta_2) &= (1+n_2)^2 M_1(\zeta_1) + m_1\Big[M_1(\zeta_2) + 8m_2 + 4n_2\Big] \\
F(\zeta_1 \triangle \zeta_2) &= (1+n_2)^3 F(\zeta_1) + m_1\Big[F(\zeta_2) + 6M_1(\zeta_2) + 24m_2 + 8n_2\Big] \\
Y(\zeta_1 \triangle \zeta_2) &= (1+n_2)^4 Y(\zeta_1) + m_1\Big[Y(\zeta_2) + 8F(\zeta_2) + 24M_1(\zeta_2) + 64m_2 + 16n_2\Big]
\end{aligned}$$

We have the following conjecture for any integer $\alpha \geq 2$ and $Z^\alpha(\zeta) = \sum_{v \in V(\zeta)} d_\zeta(v)^\alpha$.

Conjecture 11. Let $\zeta_1(n_1, m_1)$ and $\zeta_2(n_2, m_2)$ be two graphs, then for any integer $\alpha \geq 2$, the Z^α of the edge corona product $\zeta_1 \triangle \zeta_2$ is

$$\begin{aligned}
Z^\alpha(\zeta_1 \triangle \zeta_2) = (1+n_2)^\alpha Z^\alpha(\zeta_1) + m_1\Big[&Z^\alpha(\zeta_2) + x_{\alpha-1} Z^{\alpha-1}(\zeta_2) + x_{\alpha-2} Z^{\alpha-2}(\zeta_2) + \dots \\
&+ x_2 Z^2(\zeta_2) + x_1 Z^1(\zeta_2) + \alpha \cdot 2^\alpha m_2 + 2^\alpha\ n_2\Big],
\end{aligned}$$

where $x_i's$ are some positive integers.

4. Conclusion

In this paper, we have discussed a very famous graph operation called edge corona product and derived some degree base topological indices of this product. We proposed its applications for silicate networks. In Tables {1}-{4}, we calculated these results for n-dimensional chains, cyclic silicate, triangular and double triangular snake structures as corollaries. In the future, one can investigate other topological indices for the edge corona product graphs and also one can work on the new graph operations with the applications and can find their topological invariants. We are trying to get generalized Peterson graph and Dutch Windmill graph and other well known structures by the help of graph operations as a future project. At the end, we presented a conjecture.

Conflict of interest. The authors declare that they have no conflict of interest.

Data Availability. Data sharing is not applicable to this article as no data sets were generated or analyzed during the current study.

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