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Research Article

A construction of cospectral signed line graphs

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Abstract: For an ordinary graph G, we compute the eigenvalues and the eigenspaces of the signed line graph $\mathcal{L}(\ddot{G})$, where $\ddot{G}$ is obtained from G by inserting a negative parallel edge between every pair of adjacent vertices. As an application, we prove that if G and H share the same vertex degrees, then $\mathcal{L}(\ddot{G})$ and $\mathcal{L}(\ddot{H})$ share the same spectrum. To the best of our knowledge, this construction does not follow the line of any known construction developed for either graphs or signed graphs. Among the other consequences, we emphasize that $\mathcal{L}(\ddot{G})$ is integral (i.e., its spectrum consists entirely of integers), which means that a construction of integral signed graphs has been established simultaneously.

Keywords: vertex degree, signed line graph, adjacency matrix, eigenspace, cospectral signed graphs, integral spectrum.

AMS Subject classification: 05C50, 05C22

1. Introduction

We consider finite undirected signed graphs without loops. Moreover, the title of the paper refers to signed graphs having no multiple (or parallel) edges neither. To construct them, in an intermediate step, we allow the existence of at most two parallel edges, one positive the other negative; the details are given in the end of this section. Throughout the text, when say a 'graph' (without the preceding word 'signed'), we always mean an ordinary finite undirected graph without loops or multiple edges.
Two graphs are *cospectral* if they are not isomorphic, but share the same spectrum. For signed graphs, isomorphism is usually combined with switching equivalence to the more general concept of switching isomorphism of signed graphs. Accordingly, signed graphs are *cospectral* if they are not switching isomorphic, but share the same spectrum. It is well-known that, in general, graphs are not determined by their multiset of eigenvalues of the adjacency matrix or any other prescribed graph matrix, but it is conceivable that almost all graphs have this property – an assertion known

as the Haemers conjecture [15]. Accordingly, constructing cospectral graphs appears as a long-standing and challenging problem. A majority of constructions starts with the adjacency matrix of a single graph and transforms it into a similar matrix that features as the adjacency matrix of a non-isomorphic graph. In graph terminology, such a transformation is interpreted as a prescribed edge perturbation between two or more vertex subsets. Notable routines in this direction are the Seidel switching [19], its generalization developed by Godsil and McKay (the GM-switching) [13] and the more recent switching developed by Wang, Qiu and Hu (the WQH-switching) [17]. More constructions that follow the same idea can be found in [2, 11, 12]; slightly different approaches are offered by Butler for bipartite graphs [9] and Cvetković [10] for trees. The GM-switching and the WQH-switching are extended to signed graphs in [4]. More constructions of cospectral signed graphs can be found in [5, 24].

It occurs that, in the framework of signed graphs, there is a simple and surprising construction developed on the basis of signed line graphs. To the best of our knowledge, there is no similar construction for ordinary graphs. In the case of regular signed graphs it appears in [22, 24], and in this paper we extend it to pairs of signed graphs sharing the same vertex degrees. The result is given in a wider context in which we explicitly compute the eigenvalues and the corresponding eigenspaces of signed line graphs in question. It appears that every eigenvalue is an integer, and so we simultaneously establish a construction of integral signed graphs.

In the remainder of this section we give all necessary notions, terminology and notation. The main contribution is reported in Section 2. Some consequences and further developing are separated in Section 3.

A *signed graph* $\dot{G}=(G,\sigma)$ consists of an underlying graph $G=(V,E)$ with a *signature function* σ that maps the edge set E into $\{1,-1\}$. The edges mapped to 1 are *positive*, those mapped to -1 are *negative*, and together they comprise the edge set of $\dot{G}$. A graph is interpreted as a signed graph in which all edges are positive; it is recognized in the text by the absence of a dot symbol. The number of vertices and the number of edges are called the *order* and the *size* of $\dot{G}$.

If the order of $\dot{G}$ is n, then its *adjacency matrix* $A_{\dot{G}}$ is the $n\times n$ vertex-vertex $\{0,1,-1\}$-matrix which is obtained from the adjacency matrix of its underlying graph by reversing the sign of all 1s which correspond to negative edges. By the *eigenvalues* and the *spectrum* of $\dot{G}$, we mean the eigenvalues and the spectrum of $A(\dot{G})$. The *Laplacian matrix* of $\dot{G}$ is $L(\dot{G})=D(\dot{G})-A(\dot{G})$, where $D(\dot{G})$ is the diagonal matrix of vertex degrees (in G or $\dot{G}$, all the same).

Signed graphs $\dot{G}$ and $\dot{H}$ are *switching equivalent* if $\dot{H}$ is obtained by selecting a vertex subset of $\dot{G}$ and reversing the sign of every edge with exactly one end in the selected subset. Switching equivalent signed graphs share the same spectrum since the corresponding adjacency matrices are similar.

In this paper, we use the definition of signed line graphs that can be found in [7, 22–24]. This concept is tailored to spectral graph theory and differs in sign from that of [26]; a comprehensive comparison is given in [8]. So, for a signed graph $\dot{G}$ we introduce the vertex-edge orientation $\eta\colon V(G)\times E(G)\longrightarrow\{0,1,-1\}$ formed by obeying the following rules: $\eta(i,jk)=0$ if $i\notin\{j,k\}$, $\eta(i,ij)\in\{1,-1\}$ and $\eta(i,ij)\eta(j,ij)=$

$-\sigma(ij)$. Accordingly, one may randomly choose $\eta(i, ij)$ to be either 1 or -1, but $\eta(j, ij)$ is then fixed by $\sigma(ij)$; this will be used in the forthcoming Figure 1. The vertex-edge incidence matrix B_η is the matrix whose (i, e) entry is $\eta(i, e)$. The adjacency matrix of a signed line graph $\mathcal{L}(\dot{G})$ is

$$A(\mathcal{L}(\dot{G})) = B_\eta^\intercal B_\eta - 2I \tag{1.1}$$

where I is the identity matrix. A signed line graph depends on the orientation η, but different orientations produce switching equivalent signed line graphs. Also, switching equivalent signed graphs produce switching equivalent signed line graphs (see [8]).
The Laplacian matrix can be derived as the row-by-row product of the matrix B_η with itself:

$$L(\dot{G}) = B_\eta B_\eta^\intercal. \tag{1.2}$$

Regardless of the orientation η chosen, we get the same $L(\dot{G})$.
In a signed graph, two parallel edges (i.e., two edges between the same pair of vertices) form a cycle of length 2 called a *digon*. A digon is positive if its edges have the same sign, and negative if they differ in sign. It follows that the existence of a positive digon in $\dot{G}$ implies the existence of parallel edges in its signed line graph. On the contrary, a negative digon produces non-adjacent vertices. A signed graph which allows parallel edges if and only if they form negative digons is called by Zaslavsky [26] a *simply signed graph*. Accordingly, $\mathcal{L}(\dot{G})$ has no multiple edges if and only if $\dot{G}$ is a simply signed graph. For a graph G, we denote by $\ddot{G}$ the *signed doubled graph* obtained from G by replacing every edge with a negative digon.
The adjacency matrix of a simple signed graph coincides with the adjacency matrix of a signed graph obtained by removing all parallel edges. The definition of the corresponding Laplacian matrix follows the same line. Intuitively, a positive and a negative edge between the same pair of vertices cancel each other.

2. Results

We exploit the idea that, for a matrix M, the matrices $MM^\intercal$ and $M^\intercal M$ are positive semidefinite and share the same non-zero eigenvalues. Both properties are well-known, but for the sake of completeness, we include short proofs. Positive semidefiniteness follows from $\mathbf{y}^\intercal MM^\intercal\mathbf{y} = (M^\intercal\mathbf{y})^\intercal(M^\intercal\mathbf{y}) \geq 0$, for every vector $\mathbf{y}$ of feasible size. If λ is a non-zero eigenvalue of $MM^\intercal$ associated with an eigenvector $\mathbf{x}$, then we have $MM^\intercal\mathbf{x} = \lambda\mathbf{x}$, which implies $M^\intercal M(M^\intercal\mathbf{x}) = \lambda M^\intercal\mathbf{x}$, and this gives the latter assertion. We have simultaneously proved that the eigenspace for λ in $M^\intercal M$ is determined by the eigenspace for the same eigenvalue in $MM^\intercal$.
By setting $M = B_\eta$, we find that the spectrum of $\mathcal{L}(\dot{G})$ is bounded below by -2 and $L(\dot{G})$ is positive semidefinite, for every signed graph $\dot{G}$. We proceed with two lemmas.

Lemma 1. *Given a graph G without isolated vertices, let $\ddot{G}$ be the corresponding signed doubled graph. The following statements hold true:*

(i) The Laplacian matrix of $\ddot{G}$ is positive semidefinite.

(ii) If B_η is a vertex-edge incidence matrix of $\ddot{G}$, then $\ker(B_\eta^\intercal) = \mathbf{0}$.

Proof. (i): The Laplacian matrix $L(\ddot{G})$ has vertex degrees on the main diagonal and zeros outside this diagonal. Since G has no isolated vertices, the main diagonal has no zero entries which gives the desired result.
(ii): If $\mathbf{x} \in \ker(B_\eta^\intercal)$, then $\mathbf{x} \in \ker(B_\eta B_\eta^\intercal)$. The identity (1.2), in conjunction with item (i), leads to $\mathbf{x} = \mathbf{0}$. □

Lemma 2. *Let G be a graph without isolated vertices. The dimension of the eigenspace $\mathcal{E}(-2)$ in $\mathcal{L}(\ddot{G})$ is $2m - n$, where n and m are the order and the size of G, respectively.*

Proof. The $2m \times 2m$ matrix $B_\eta^\intercal B_\eta$ and the $n \times n$ matrix $B_\eta B_\eta^\intercal$ share the same non-zero eigenvalues. By Lemma 1(i), the latter matrix has n non-zero eigenvalues, which means that the multiplicity of zero in the former matrix is $2m - n$. The desired result follows from the identity (1.1). □

We now compute the eigenvalues distinct from -2 and the corresponding eigenvectors of a signed doubled graph.

Theorem 1. *Given a graph G without isolated vertices, let $\ddot{G}$ be the corresponding signed doubled graph. Suppose that the vertices $1, 2, \ldots, n$ of G are arranged in a non-increasing order according to their degrees $d_1, d_2, \ldots, d_n$. The eigenvalues distinct from -2 of $\mathcal{L}(\ddot{G})$ are $2(d_i - 1)$, for $1 \leq i \leq n$. An eigenvector associated with $2(d_i - 1)$ is $B_\eta^\intercal \mathbf{e}_i$, where B_η is a vertex-edge incidence matrix of $\ddot{G}$, and $\mathbf{e}_i$ is the ith vector of the canonic basis of $\mathbb{R}^n$.*

Proof. The Laplacian matrix of $\ddot{G}$ is the diagonal matrix $\mathrm{diag}(2d_1, 2d_2, \ldots, 2d_n)$, and therefore the corresponding eigenvalues $2d_1, 2d_2, \ldots, 2d_n$ are associated with eigenvectors $\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n$, respectively. Using the discussion given at the beginning of this section together with (1.1) and (1.2), we find that the eigenvalues of $\mathcal{L}(\ddot{G})$ are $2(d_1 - 1), 2(d_2 - 1), \ldots, 2(d_n - 1)$. The proof is completed by taking into account that $A(\mathcal{L}(\ddot{G}))$ shares the eigenvectors with $B_\eta^\intercal B_\eta$. □

We proceed with the eigenspace for -2.

Theorem 2. *Given a graph G without isolated vertices, let $\ddot{G}$ be the corresponding signed doubled graph. Then*

$$\mathcal{E}(-2) = \ker(B_\eta),$$

where $\mathcal{E}(-2)$ is the eigenspace for -2 in $A(\mathcal{L}(\ddot{G}))$ and B_η is a vertex-edge orientation of $\ddot{G}$.

Proof. Let $\mathbf{x} \in \mathcal{E}(-2)$. From (1.1), we have $B_\eta^\intercal B_\eta \mathbf{x} = \mathbf{0}$. By virtue of Lemma 1(ii), $B_\eta^\intercal$ has a trivial kernel, which implies $B_\eta \mathbf{x} = \mathbf{0}$, that is $\mathbf{x} \in \ker(B_\eta)$.
Suppose now that $\mathbf{x} \in \ker(B_\eta)$. We compute

$$A(\mathcal{L}(\ddot{G}))\mathbf{x} = \big(B_\eta^\intercal B_\eta - 2I\big)\mathbf{x} = B_\eta^\intercal B_\eta\mathbf{x} - 2I\mathbf{x} = \mathbf{0} - 2\mathbf{x},$$

giving $\mathbf{x} \in \mathcal{E}(-2)$. □

Here is the main application of the previous results.

Theorem 3. *Let G and H be graphs of order n that share the same vertex degree sequence $d_1, d_2, \dots, d_n$. The signed line graphs $\mathcal{L}(\ddot{G})$ and $\mathcal{L}(\ddot{H})$ share the same spectrum.*

Proof. Without loss of generality, we may suppose that $d_1 \geq d_2 \geq \cdots \geq d_n$. If k is the largest integer such that $d_k > 0$, then by Lemma 2 and Theorem 1 $\mathcal{L}(\ddot{G})$ and $\mathcal{L}(\ddot{H})$ share the spectrum consisting of -2 with multiplicity $\sum_{i=1}^{k} d_i - k$ (the sum is twice the number of edges in G) and $2(d_i - 1)$, for $1 \leq i \leq k$. □

Although there exist non-isomorphic signed graphs that produce switching isomorphic signed line graphs, from a still unpublished manuscript [20] we know that if G and H of the previous theorem are non-isomorphic, then $\mathcal{L}(\ddot{G})$ and $\mathcal{L}(\ddot{H})$ are switching non-isomorphic.
We conclude the section with an example.

Example 1. The smallest pair of non-isomorphic graphs that share the same vertex degrees are the graphs G and H of order 5 illustrated in Figure 1. The corresponding signed doubled graphs and their signed line graphs are in the same figure. This example also shows that vertex degrees of the corresponding signed line graphs may differ. According to Lemma 2 and Theorem 1, their common spectrum is $[4, 2^3, 0, (-2)^5]$ (an exponent denotes the multiplicity).

3. Other consequences of Lemma 2 and Theorem 1

Integral signed graphs. Integral signed graphs are investigated in [6, 21, 25]. In addition, there is an extensive literature concerning integral graphs, not listed here. It occurs that integral graphs are a rare phenomenon. For example, it is established in [1] that only 150 connected graphs with at most 10 vertices are integral; less than 0.002% of the total number. Also, only 13 connected regular graphs of vertex degree 3 are integral [18]. On the other hand, every graph gives rise to an integral signed graph – the line graph of its signed double.

Inertia. The *inertia* of a signed graph is an integer triple specifying the numbers of positive, negative and zero eigenvalues of the adjacency matrix. For a graph G without isolated vertices, the inertia of $\mathcal{L}(\ddot{G})$ is $(n - d_1, 2m - n, d_1)$, where n, m and d_1 are the order, the size and the number of pendant vertices (i.e., vertices of degree 1) of G, respectively. Moreover, the following facts are direct consequences of Lemma 2 and Theorem 1:

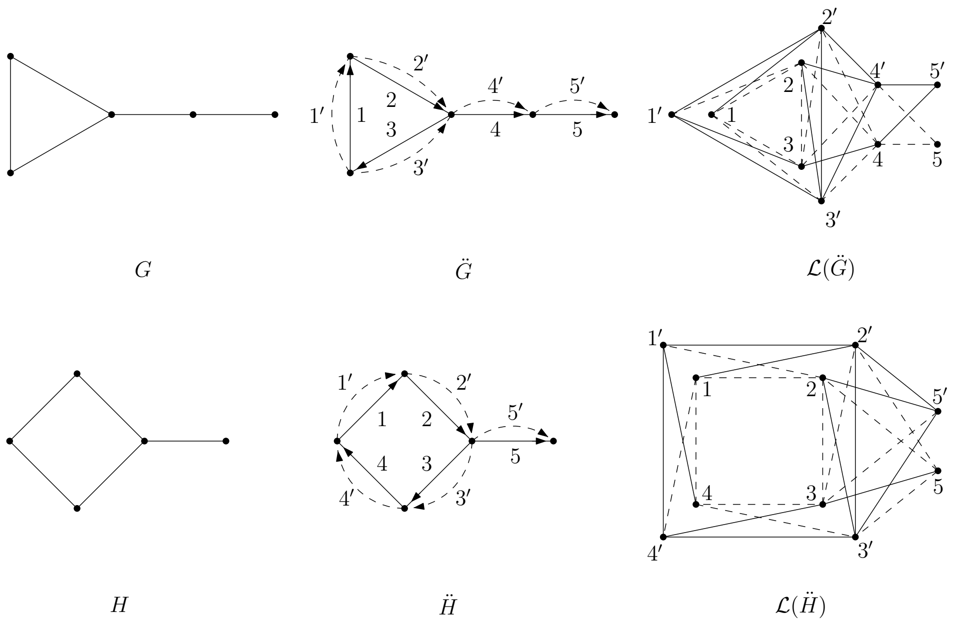

Figure 1. Graphs G and H sharing the same degree sequence, signed doubled graphs $\ddot{G}$ and $\ddot{H}$, and signed line graphs $\mathcal{L}(\ddot{G})$ and $\mathcal{L}(\ddot{H})$. For $\ddot{G}$ and $\ddot{H}$, an arrow indicates a vertex-edge orientation; for the sake of simplicity, only one orientation of each edge is given.

Fact 1. *The number -2 appears in the spectrum of $\mathcal{L}(\ddot{G})$ if an only if G is not a disjoint union of isolated vertices and isolated edges. If λ is a negative eigenvalue of $\mathcal{L}(\ddot{G})$, then $\lambda = -2$.*

Fact 2. *The signed graph $\mathcal{L}(\ddot{G})$ is singular (i.e., has 0 as an eigenvalue) if and only if G has a pendant vertex.*

Fact 3. *The signed graph $\mathcal{L}(\ddot{G})$ has exactly one positive eigenvalue if and only if G is a disjoint union of isolated vertices, isolated edges and a star $K_{1,n}$, with $n \geq 2$.*

Distinct eigenvalues. If G is a connected graph with at least two edges, then the number of distinct eigenvalues of $\mathcal{L}(\ddot{G})$ is the number of distinct vertex degrees in G plus one. In particular, $\mathcal{L}(\ddot{G})$ has exactly two distinct eigenvalues if and only if G is regular. By the pigeonhole principle, the number of distinct eigenvalues of $\mathcal{L}(\ddot{G})$ does not exceed the order of G.

Signed line graphs with spectrum in $[-2, 2]$**.** In [16], McKee and Smyth proved that the spectrum of a connected signed graph $\dot{G}$ lies in the segment $[-2, 2]$ if and only if $\dot{G}$ is a subgraph of either the so-called toral tessellation with $2n$ ($n \geq 3$) vertices or one of two particular signed graphs having 14 and 16 vertices, respectively. It is proved in [23] that a toral tessellation is in fact a signed line graph $\mathcal{L}(\ddot{C}_n)$ (where C_n is a cycle of order n), whereas the remaining two signed graphs are not signed line graphs. By taking into account Lemma 2 and Theorem 1, together with the eigenvalue interlacing, we deduce that the spectrum of a signed line graph $\mathcal{L}(\dot{G})$ lies in $[-2, 2]$ if and only if $\dot{G}$ is a subgraph of $\ddot{C}_n$, for some n, or $\mathcal{L}(\dot{G})$ is an induced subgraph of some of the two aforementioned particular signed graphs.

The least Laplacian eigenvalue of a proper simply signed graph. In [3], Belardo proved that the least Laplacian eigenvalue of a connected signed graph $\dot{G}$ (without parallel edges) is zero if and only if $\dot{G}$ is balanced, i.e., switches to its underlying graph. This statement extends to simply signed graphs with at least one negative digon.

Theorem 4. *Let $\dot{G}$ be a connected simply signed graph with at least one negative digon. Then the least Laplacian eigenvalue of $\dot{G}$ is greater than zero.*

Proof. Since $\dot{G}$ is connected, it has a connected spanning subgraph $\dot{F}$ which contains a single cycle isomorphic to a negative digon. Thus, $\dot{F}$ has n edges and, by [14, Theorem 6], each of the n eigenvalues of $\mathcal{L}(\dot{F})$ is greater than -2. If G is the underlying graph of $\dot{G}$, then $\ddot{G}$ is obtained from $\dot{G}$ by forming a negative digon for every pair of vertices joined by a single edge. Say that $\ddot{G}$ is obtained by adding exactly k edges to $\dot{G}$. By Lemma 2, the multiplicity of -2 in $\mathcal{L}(\ddot{G})$ is $m+k-n$, where m is the size of $\dot{G}$. Observing that $\dot{F}$ is obtained from $\ddot{G}$ by deleting $m+k-n$ edges and employing the eigenvalue interlacing, we deduce that in passing from $\ddot{G}$ to $\dot{F}$ every removal of a single edge decreases the multiplicity of -2 (in the spectrum of the corresponding signed line graph) by 1. Thus, the multiplicity of -2 in $\mathcal{L}(\dot{G})$ is $m-n$, meaning that exactly n eigenvalues are greater than -2, which in turn implies that every eigenvalue of the Laplacian $L(\dot{G})$ is greater than zero. □

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