Communications in Combinatorics and Optimization
Vol. xx, No. x (xxxx), pp. 1-5
https://doi.org/10.22049/cco.2026.31339.2821

CCO
Commun. Comb. Optim.

Short Note

A correction to the R_k-construction for graphs with integer Sombor index

Luca Guerrini
Department of Management, Polytechnic University of Marche,
Piazzale Martelli 8, 60121 Ancona, Italy
luca.guerrini@univpm.it

Received: 18 December 2025; Accepted: 9 July 2026
Published Online: 12 July 2026

Abstract: In this short note, we report an error in the proof of Theorem 1 of Noor A'lawiah Abd Aziz, which concerns the R_k-construction for generating graphs with integer Sombor index. We provide a corrected proof and the corresponding formula. Importantly, we verify that the main conclusion of the theorem remains valid despite this correction.

Keywords: Sombor index, topological index, graph construction

AMS Subject Classification: 05C69, 05C78

1. Introduction

For a graph G, the Sombor index of G is defined as

$$SO(G) = \sum_{uv \in E(G)} \sqrt{\deg(u)^2 + \deg(v)^2},$$

where $\deg(u)$ denotes the degree of vertex u in G. The Sombor index was introduced by Gutman [2] and has attracted considerable attention in recent years. One important research direction concerns the characterization of graphs whose Sombor indices are integers.

In a recent paper [1], Abd Aziz introduced the R_k-construction, which produces from a given graph G and an integer $k \geq 2$ a new graph $R_k(G)$ with the property that if $SO(G)$ is an integer, then $SO(R_k(G))$ is also an integer. The construction was claimed to satisfy the formula

$$SO(R_k^l(G)) = \left(\frac{k^3 + k^2}{2}\right)^l SO(G)$$

for all $l \geq 1$ and $k \geq 2$, where $R_k^l(G)$ denotes the l-fold iterated application of the R_k-construction.
In this note, we point out an algebraic error in the proof of this result and provide the correct formula. Fortunately, the main conclusion that the construction produces infinitely many graphs with integer Sombor index remains valid.

2. The R_k-construction

We briefly recall the R_k-construction from [1]. Given a graph G with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and an integer $k \geq 2$, the graph $R_k(G)$ is constructed as follows: Consider k copies of G, denoted $G^1, G^2, \ldots, G^k$, where

$$V(G^i) = \{v_1^i, v_2^i, \ldots, v_n^i\}, \quad E(G^i) = \{v_r^i v_s^i \mid v_r v_s \in E(G)\}$$

for $i = 1, 2, \ldots, k$. Then

$$V(R_k(G)) = \bigcup_{i=1}^{k} V(G^i)$$

and

$$E(R_k(G)) = \bigcup_{i=1}^{k} E(G^i) \cup \{v_r^i v_s^j \mid i \neq j, v_r v_s \in E(G)\}.$$

In other words, $R_k(G)$ consists of k copies of G, with each edge of G replaced by a complete bipartite graph $K_{k,k}$ in $R_k(G)$ (where the two parts correspond to the k copies of each endpoint). An important observation is that for any vertex $v_j^i \in V(R_k(G))$, we have

$$\deg_{R_k(G)}(v_j^i) = k \cdot \deg_G(v_j).$$

3. The error in Theorem 1

Theorem 1 of [1] claims that for each $l \geq 1$ and $k \geq 2$,

$$SO(R_k^l(G)) = \left(\frac{k^3 + k^2}{2}\right)^l SO(G).$$

The error occurs in the proof when computing $SO(R_k(G))$. Let us carefully recompute this quantity. We partition the edges of $R_k(G)$ into two types:

(i) Edges within a single copy: $v_r^i v_s^i$ for some i and $v_r v_s \in E(G)$.

(ii) Edges between different copies: $v_r^i v_s^j$ for $i \neq j$ and $v_r v_s \in E(G)$.

For edges of type (i), we have

$$\sum_{i=1}^{k}\sum_{v_rv_s\in E(G)}\sqrt{\deg^2_{R_k(G)}(v_r^i)+\deg^2_{R_k(G)}(v_s^i)} = k\sum_{v_rv_s\in E(G)}\sqrt{k^2\deg^2_G(v_r)+k^2\deg^2_G(v_s)}$$
$$= k^2\sum_{v_rv_s\in E(G)}\sqrt{\deg^2_G(v_r)+\deg^2_G(v_s)}$$
$$= k^2\cdot SO(G).$$

For edges of type (ii), we need to count carefully. For each edge $v_rv_s\in E(G)$ and each ordered pair of distinct indices (i,j) with $i\neq j$, there is an edge $v_r^iv_s^j$ in $R_k(G)$. The number of such ordered pairs is $k(k-1)$. Therefore,

$$\sum_{\substack{1\le i,j\le k\\ i\neq j}}\sum_{v_rv_s\in E(G)}\sqrt{\deg^2_{R_k(G)}(v_r^i)+\deg^2_{R_k(G)}(v_s^j)}$$
$$= k(k-1)\sum_{v_rv_s\in E(G)}\sqrt{k^2\deg^2_G(v_r)+k^2\deg^2_G(v_s)}$$
$$= k^2(k-1)\cdot SO(G).$$

Adding both contributions, we obtain

$$SO(R_k(G)) = k^2\cdot SO(G)+k^2(k-1)\cdot SO(G) = k^2(1+k-1)\cdot SO(G) = k^3\cdot SO(G).$$

This leads us to the correct version of Theorem 1.

Theorem 1 (Corrected). *For each $l\ge 1$ and $k\ge 2$,*

$$SO(R_k^l(G)) = k^{3l}\cdot SO(G).$$

Proof. We have shown that $SO(R_k(G)) = k^3\cdot SO(G)$. The general result follows by induction on l. For the base case $l=1$, this is exactly what we proved above. For the inductive step, assume $SO(R_k^{l-1}(G)) = k^{3(l-1)}\cdot SO(G)$. Then $SO(R_k^l(G)) = SO(R_k(R_k^{l-1}(G))) = k^3\cdot SO(R_k^{l-1}(G)) = k^3\cdot k^{3(l-1)}\cdot SO(G) = k^{3l}\cdot SO(G)$. □

Despite the error in the explicit formula, the main conclusion of [1] remains valid:

Corollary 1. *If G is a graph with integer Sombor index, then for each $l\ge 1$ and $k\ge 2$, $R_k^l(G)$ is a graph with integer Sombor index.*

Proof. Since k^{3l} is an integer for all integers $k\ge 2$ and $l\ge 1$, we have that $SO(R_k^l(G)) = k^{3l}\cdot SO(G)$ is an integer whenever $SO(G)$ is an integer. □

Thus, the R_k-construction does indeed provide a method for generating infinitely many families of graphs with integer Sombor index, as originally claimed.

4. Remarks on related indices

The paper [1] also states similar results for the Randić index and reciprocal Randić index in Theorem 2. We briefly verify these claims. For the reciprocal Randić index, defined as

$$RR(G) = \sum_{uv \in E(G)} \sqrt{\deg(u) \cdot \deg(v)},$$

a similar calculation shows that

$$RR(R_k(G)) = k^3 \cdot RR(G),$$

and thus $RR(R_k^l(G)) = k^{3l} \cdot RR(G)$. For the Randić index, defined as

$$R(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\deg(u) \cdot \deg(v)}},$$

the computation is different due to the reciprocal. In $R_k(G)$, each vertex corresponding to a vertex $u \in V(G)$ has degree $k \deg_G(u)$. Hence, for each edge corresponding to $uv \in E(G)$, the Randić weight becomes

$$\frac{1}{\sqrt{(k \deg_G(u))(k \deg_G(v))}} = \frac{1}{k} \frac{1}{\sqrt{\deg_G(u) \deg_G(v)}}.$$

For the edges within the copies, there are k copies of G. Therefore, their total contribution is

$$k \sum_{uv \in E(G)} \frac{1}{k} \frac{1}{\sqrt{\deg_G(u) \deg_G(v)}} = R(G).$$

For the edges between different copies, there are $k(k-1)$ ordered pairs of distinct copies. Thus, their total contribution is

$$k(k-1) \sum_{uv \in E(G)} \frac{1}{k} \frac{1}{\sqrt{\deg_G(u) \deg_G(v)}} = (k-1)R(G).$$

Consequently,

$$R(R_k(G)) = R(G) + (k-1)R(G) = kR(G),$$

and, by induction,

$$R(R_k^l(G)) = k^l R(G)$$

for all $l \geq 1$. Therefore, the claim in [1] that

$$R(R_k^l(G)) = \left(\frac{k+1}{2}\right)^l R(G)$$

is incorrect, and the correct multiplicative factor is k^l.

Acknowledgements: The author is grateful to the referee for the careful reading and constructive suggestion, which helped improve the accuracy of the manuscript.

Conflict of Interest: The authors declare that they have no conflict of interest.

Data Availability: Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

References

[1] N. A'lawiah Abd Aziz, *A note on graphs with integer Sombor index*, Commun. Comb. Optim. **11** (2026), no. 1, 205–209. https://doi.org/10.22049/cco.2024.29474.2013.

[2] Ivan Gutman, *Geometric approach to degree-based topological indices: Sombor indices*, MATCH Commun. Math. Comput. Chem. **86** (2021), no. 1, 11–16.